Problems in the answer key:

Problem 8.16: The answer key did not account for the thermal noise generated by the resistor.

Problem 8.18: The answer key did not account for the thermal noise generated by the resistor on the non-inverting terminal. It also calculates $V_{\text{onoise, RMS}}$ by adding all the noise component directly together, which is wrong.

Problem 6: Noise at the output has a high-pass behavior.

For Alin Dosa:

He made all the mistakes mentioned above.
He did not turn in SPICE netlist for the last problem.
He did not get the equation for $V_{\text{onoise}(f)}$ and he forgot to integrate $V_{\text{onoise}(f)}$ to get $V_{\text{onoise,RMS}}$ in SPICE simulation for the last problem.
Homework 2 Summary

Problem 1 (10/10)
Both Tao and I did it right. This problem helps me to get a clear view of how to use the input-referred noise model and how to validate it.

Problem 2 (10/10)
Both Tao and I did it right.

Problem 3 (10/10)
Both Tao and I did it right.

Problem 4 (9/10)
I think in the last question of this problem, Tao made a mistake. I think to make the output noise small, we should decrease the feedback resistor instead of the capacitor.

Problem 5 (30/30)
Great work

Problem 6 (25/30)
The equation seems correct. But lacks spice result. I think it would be better if he put the exact value of the parameters in the VoniseRms and compare it to the spice result.
1. This problem shows that as the input resistance of an amplifier approaches infinity, the noise model cannot be simplified. This result seems counterintuitive because when $R_{in}$ is infinity, it is merely an open circuit and should not contribute to the output noise. However, a closer analysis does show this. The use of the limit operator is imperative to the analysis.

2. A tricky part of the problem is seen when realizing that the RMS squared voltages are converted to RMS voltages by taking the square root of the sum of the squares. The analysis Luis performed is very similar to the one that I followed. It is of interest to note that the factor of 49 in the flicker noise estimation also appears in his analysis. This factor comes from the analysis leading to equation 8.56.

3. This problem was one of the most straightforward problems in the assignment. Realizing that the resistor noise ($4kT/R$) and diode shot noise are the only noise contributors, the analysis is rather simple. The DC current flows through the diode and resistor in series. The voltage drop over the resistor is given by the supply and Zener diode voltage.

4. The analysis in this section is done through superposition of each of the 3 resistor’s voltage or current noise model. In the discussion of the methods to reduce noise, it is noted that increasing capacitance and removing the 1k resistor can reduce the noise. Another method to reduce the noise includes reducing the values of the resistors themselves. This way the operation of the circuit isn’t modified as would be if the 1k resistor is removed.

5. Superposition is once again used to determine the value of the noise. In contrast to my submission, Luis has found that the noise converges to an infinite value. This does not seem to be a realistic value for noise, especially across all frequency values.

6. This circuit is also solved in a relatively simple fashion as only noise from the resistor is factored in. A finite estimate for the noise is evaluated and similar to the $kT/C$ noise voltage, there exists a $kT/L$ noise current relationship in this circuit.
Problem 1.
Work looks good; realizing it’s a trick question (Baker’s statement is false) is the hard part.

Problem 2.
Good job!

Problem 3.
You can compute the small signal resistance of the diode very easily (take derivative of diode current with respect to voltage). Saying 1k << rz is not true, because if you calculate rz you get 6 ohms; can’t always trust the solutions manual.

Problem 4.
Noise due to R2 doesn’t add with all the other current noise sources, therefore your final answer was wrong. Again, you can’t always trust the solutions manual. Because of the virtual ground at the inverting terminal of opamp, the noise from R2 will directly couple into the output of the circuit.

Problem 5.
If the opamp has infinite gain, then that implies that the op amp also has infinite bandwidth (Ideal). When integrating the noise due to the input referred noise, you’re integrating a high pass function from 0 to infinity therefore without a fixed bandwidth the total integrated noise will go to infinity (input and output). The problem wasn’t worded well, so I only took off one point.

Problem 6.
If you solve the noise using a current source for the resistor noise, the problem becomes very easy, and you can convert to voltage using R. The problem doesn’t state that you must use voltage or current, it just says find the noise.
HW2 Summary

General Summary of assignment (what was learned)

- This homework provided a good basis in theory of noise analysis and an overview of basic circuit configurations. More problems involving SPICE simulations would have been beneficial so as to see how well theory agrees with simulation
- This homework provided a good overview of op amp amplifier noise analysis.
- I think inclusion of at least one design problem might be beneficial (e.g. tradeoffs between $kT/C$ noise, bandwidth, etc.)
- Several problems in this homework provided good examples of how noise analysis can be greatly simplified (i.e. not using rigorous methods of integration, etc.) with a few simple assumptions about the circuit or previously proven equations (e.g. use of NEB for low-pass filters, noise equations for op amp amplifier circuits).
- Problem 6 was interesting as despite being the same circuit type (low-pass filter) as the circuit used to derive $kT/C$ noise, the results and implications are completely different.
- Overall, the homework provided a good overview of noise analysis of many circuits, including diode circuits, op amp circuits, and passive filters.

Notes on Graded Homework (Faisal’s)

- There were minor errors on Problem 5 ($f_{3dB}$ and NEB calculation). Analysis method was correct.
- Problem 6 schematic did not match thermal noise equation. $4KTR$ vs. $4KT/R$ for voltage vs. current noise sources.
- Calculation of output RMS noise was incorrect, as shown by comparison to SPICE simulations. A more rigorous calculation of output noise and then integration to RMS noise yields an equation that produces much better results.
- Perhaps mention ramifications of input RMS noise equation (i.e. as $f \to \infty$, input noise $\to \infty$. Compare this to low frequency noise where frequency is less relevant and gain is $\sim 1$
Homework 1 Summary

Grade: 98/100

Comments for my classmate – Gregory Erwin

- Problem 1
  - You should clearly state the result, i.e., that the output noise cannot be modeled only by using an input referred noise voltage

- Problem 2
  - OK

- Problem 3
  - Problem was asking for PSD, so the result should have the units of V²/Hz

- Problem 4
  - OK

- Problem 5
  - OK

- Problem 6
  - For the calculation you picked a 1H inductor; I suggest you pick a smaller inductor for a more practical case.

What I learned:

- Calculate the output noise voltage, and refer it back to the input as a voltage and current input referred noise
- Understand better how flicker noise and thermal noise come into play in an amplifier
- Role of an LC network as opposed to an RC network in terms of input referred noise as well as output noise
- I realized that I might have made a few mistakes on problems 8.18 and problem #6 (I did not properly calculate the output noise)
1. Baker 8.8
   I got a similar result. It seems the question was worded incorrectly.
   
   **Points awarded: 10/10**

2. Baker 8.14
   The setup for $C_F$ was correct, but the resulting value was off by a factor of 100.
   
   **Points awarded: 9/10**

3. Baker 8.16
   In your small-signal representation, you should have AC ground where you have 9 V.
   
   **Points awarded: 9/10**

4. Baker 8.18
   You seem to be missing the output noise due to $V_{in\,oise}(f) = 9 \, nV/\sqrt{Hz}$ and $I_{in\,oise}(f) = 0.85 \, pA/\sqrt{Hz}$ from p. 261 in Baker.
   
   **Points awarded: 9/10**

5. The solution doesn’t completely encapsulate the frequency dependence and the transfer function. The units seem off when finding the PSD.
   
   **Points awarded: 28/30**

6. I got the same result.
   
   **Points awarded: 30/30**
When calculating noise that is scaled by the complex transfer function of a system, only the magnitude information is important, since the phase of a random signal is meaningless. In other words, given an arbitrary complex transfer function, calculate the output noise voltage as follows:

\[ V_{Noise,Out}(f) = V_{Noise,In}(f) \cdot |H(f)|^2 \]

Note that the magnitude squared value of a complex transfer function can be calculated using the complex conjugate, i.e.:

\[ |H(f)|^2 = H(f) \cdot H(f)^* \]

Thus, it is more correct to describe the RMS output voltage of a first order lowpass filter as:

\[ V_{Noise,Out,RMS}(f) = V_{Noise,In,RMS}(f) \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}} \]

Instead of:

\[ V_{Noise,Out,RMS}(f) = V_{Noise,In,RMS}(f) \frac{1}{1 + \frac{f}{f_c}} \]
Things I learned from this assignment:

When adding noise sources in parallel, it is better to use noise currents than noise voltages, so convert the noise to the 4KT/R current whenever there’s a node with several resistors attached.

That said, one thing I learned from grading Adam’s homework is that there are multiple ways to accomplish the same goal. In particular, adding noise voltages in parallel can be accomplished with superposition, and has the advantage of being a much more intuitive method than the one described in the book, in part due to the following note:

It’s important to pay attention to node voltages when determining the path of noise current. For example, whenever an op amp holds a node at zero volts due to the virtual short between its terminals, it’s likely that the noise current due to the input resistor will only travel through the feedback network because the far end of the input resistor is tied to the source voltage, which gets shorted whenever you analyze noise. If the voltage at both ends of a resistor are the same, no current (even noise) will flow through it.

In LT spice, I learned how to perform noise simulations, something I had never done before, and I became more comfortable with using voltage controlled voltage supplies. Lastly, I discovered that CTRL + left clicking the name of a plotted trace allows you to integrate that function over some bandwidth, which is important for determining RMS noise from PSD.

Comments for Adam:
As indicated on the graded homework, you can approximate the RMS value by integrating over a definite integral of sufficiently large bandwidth, and in that way avoid crashing LT spice.
I checked Saad Bin Nasir’s assignment. He scored 95/100.

The first problem familiarized students with the input-referred noise model and demonstrated that under specific conditions, both input noise voltage and input noise current sources may not be necessary to adequately characterize system noise.

The second, fourth and fifth problems required students to model op amp noise, as well as individual component noise, and assimilate them together in a given circuit. They also helped students realize the difference between the several noise-related parameters (such as input and output referred noise, RMS noise voltage, noise PSD) and how to derive one from the other.

The third problem required students to understand a different kind of noise present in diodes, shot noise, and to apply their understanding to demonstrate how this noise affects the output noise voltage.

Problem 6 was particularly interesting due to the fact that it required going back to the elementary theory of noise and then deriving the output noise of an inductor-resistor combination. The verification with SPICE also served to demonstrate how to simulate the noise of circuits.

Overall, Saad demonstrated a good understanding of the concept and the theory behind noise. The noise derivation in the sixth problem was particularly impressive, although he did make some minor mistakes in the later portions, which could probably be attributed to carelessness.
This follow up HW was necessary after going through the literature of Ch-8 of the book.

Q#1 has helped me in understanding the mathematical equivalence and drawing insights from these mathematical expressions. For example it was necessary in Q#1 for Rs to be zero for the answer to be valid.

Q#2 guides in the complete derivation of the noise in an op-amp circuit. This helps me greatly in developing skill to get the noise in our project circuits.

Q#3 is necessary to understand the generation of shot noise as the main focus in noise is usually on flicker or thermal noise in majority of the designs. Shot noise analysis become necessary when we have current mode devices in the circuit like a diode.

Q#4 and Q#5 are equivalent in terms of methodology. Q#4 focuses on the numerical nature of solving noise in op-amp base circuits. It helps in making effective mathematical assumptions like usage of Noise Equivalent Bandwidth in a single pole role off systems. It addresses different trade-offs we can go for in like noise shaping through low pass filtering at the cost of BW and speed or decrease of resistance values at the cost of increased power consumption.

Q#5 seems easy but it presents a dual of an RC system. It provides an example where Noise Equivalent Bandwidth specification cannot be provided but at the same time we can follow the same procedure and obtain the HP response of an RL system. Here the input and output referred noise (RMS) are equivalent below f3dB but change after that as the output noise increases (RMS) due to zero in the transfer function.
HW2 Summary

1. This HW was about identifying, modelling, and computing noise parameters of a given circuit. Questions checked the basic noise concepts applied to OPAMP based circuits.

2. Impact of noise contribution from various sources and types over a band of interest was seen in question 5. The thermal noise component dominates over 1/f noise as bandwidth increases. Thus for rough estimates, 1/f noise can be neglected in such cases.

3. The last question demonstrates how inductors give response complementary to capacitors, and thus estimating noise performance of inductor based circuits becomes easier.

4. In general, using capacitors in feedback and reducing resistor values can help lowering rms noise at the output.

I graded Peter McMenamin. The only mistake I found was that in 4th problem, he linearly added rms values of noise instead of adding mean squared values and then taking square root to get total rms noise.
Comments on Siwei Wang’s Homework #2

Tao Wang

The author has good understanding of electrical noise and the calculation of total electrical noise in analog circuits.

The author understands well how 1/f noise (FNN) and thermal noise contribute the total noise, and the resultant bandwidth.

The kT/C noise originally comes from the resistor’s thermal noise. However, the capacitor value C determines the bandwidth of the R-C series/parallel circuit. Looks like the author duplicates the resistor thermal noise by including both 4kTR and kT/C.

The final problem to some extent requires discussion and close thought of the bandwidth due to the existence of the inductor L. The author assumes a high-frequency of 10GHz which I don’t think very reasonable and promising.
Problem 1.
As long as there is a non-zero source resistance, the output noise cannot be adequately modeled using a single input-referred noise source.

Problem 2.
To minimize the noise contribution from $kT/C$, add up $kT/C$ and the other noise contributors in power domain and set equal to the allowable noise. Solve for $C$. With a given BW, $R$ is pretty easily solved for.

Problem 3.
The big thing that I learned here is to pay attention to the small signal resistance of the diode, being a mere 5.8 ohms, because it is very small compared to the series resistor and thus dominates the small signal parallel combination of the two. The noise appears across the small signal output resistance, which is essentially the 5.8 ohm small signal resistance of the diode.

Problem 4.
This problem was a bit tricky. The key here is to pay attention to the various noise contributors, especially ones at the inputs of the amplifier. These noise contributors will be amplified by the op-amp gain of their respective inputs, either inverting or non-inverting. Furthermore, don’t forget to square the gain to put it in power domain (obviously square the noise, too). Once the aforementioned mathematical mess is made, multiply the whole shebang by the noise equivalent bandwidth. Ways to reduce noise incur tradeoffs such as smaller BW and higher power consumption.

Problem 5.
This is a pretty straightforward problem, if you make a key assumption that the circuit has a 1st order response. Strictly speaking, there is a zero that appears looking into the non-inverting terminal which would drive the unity gain frequency to infinity and thus the noise to infinity.

Problem 6.
I got slick here and felt special. In current domain the circuit has a low-pass response. I exploited this to use the noise equivalent bandwidth to cook down the math to $kT/L$. 
ECE6414 Homework 2 Summary
Siwei Wang for Lian Duan

1  [Baker 8.8]
Correct.

2  [Baker 8.14]
Correct.

Question: Why is input referred noise band limited by 2MHz (unity gain of op-amp) rather than 1MHz (frequency range of interest).

3  [Baker 8.16]
Correct.

4  [Baker 8.18]
Correct.

5 
Correct.

6 
Correct. Please show SPICE simulation code and raw output.