\[ I_M^2(f) = \frac{KF \cdot I_D^{AF}}{f \cdot (C'_{ox})^2 LW} + \frac{8kT}{3} \cdot g_m \]

\[ V_{\text{noise}}^2(f) = \frac{KF \cdot I_D^{AF}}{f \cdot (C'_{ox})^2 LW \cdot g_m^2} + \frac{8kT}{3g_m} \]
1. In the following RL circuit:
   (a) What does this circuit do?
   (b) Find an expression for the input referred noise.
   (c) How does this circuit compare with an RC for the same function in terms of noise performance?
   (d) If we add a capacitor in parallel with R, how will the noise performance be affected?
   (e) Find an expression for the input referred noise with both L and C components.

Solution (1)
(a) Low-pass filter
(b) If we assume that the resistor thermal noise voltage source is located between R and ground, output noise will be as follow.

\[ V_{o,\text{noise}} = \sqrt{4KT R} \times \frac{j\omega L}{j\omega L + R} = \sqrt{4KT R} \times \frac{j\omega \left(\frac{L}{R}\right)}{1 + j\omega \left(\frac{L}{R}\right)} = \sqrt{4KT R} \times \frac{j\left(\frac{f}{f_{3dB}}\right)}{1 + j\left(\frac{f}{f_{3dB}}\right)} \]

where \( f_{3dB} = \frac{R}{2\pi L} \)

Here the output noise voltage has a high-pass filter shape: \( V_{o,\text{noise}}^2 = 4KT R \times \frac{\left(\frac{f}{f_{3dB}}\right)^2}{1 + \left(\frac{f}{f_{3dB}}\right)^2} \)

\[ V_{o,\text{noise(RMS)}}^2 = 4KT R \times \int_0^{\infty} \frac{\left(\frac{f}{f_{3dB}}\right)^2}{1 + \left(\frac{f}{f_{3dB}}\right)^2} df = 4KT R \times \int_0^{\infty} \frac{\left(\frac{f}{f_{3dB}}\right)^2 + 1 - 1}{1 + \left(\frac{f}{f_{3dB}}\right)^2} df \]

\[ = 4KT R \times \int_0^{\infty} 1 - \frac{1}{1 + \left(\frac{f}{f_{3dB}}\right)^2} df = \infty \]
(c) Since the thermal noise passes through the high-pass filter, the RMS output noise of this LR circuit becomes infinite. Therefore, in terms of noise performance, this LR low-pass filter is different from an RC low-pass filter, in which the thermal noise also passes through the low-pass filter and the output noise becomes $\sqrt{KT/C}$.

(d) With both L and C, and assuming $L/R > 1/RC$, the circuit will turn into a bandpass filter. Therefore, the noise will have a finite value.

\[
V_{o,\text{noise}} = \sqrt{4KT/R \times \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right)^{-1}}
\]

\[
V_{o,\text{noise(RMS)}}^2 = 4KT/R \times \int_0^{\infty} \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right)^{-2} d\omega
\]
2. In the following switch-capacitor circuit:
   
   (a) What is the function of this circuit and how does it work (briefly)?

   (b) Draw the timing diagram of $\Phi_1$, $\Phi_2$, and $\Phi_3$ for this circuit to operate properly.

   (c) Draw the schematic diagram of a traditional continuous-time circuit performing the same function.

   (d) Name two potential advantages and one disadvantage of this circuit in comparison with the continuous-time circuit in (c).

   (e) Find an expression for the transfer function of this circuit assuming that the clock frequency is much higher than $V_{\text{in}}$ bandwidth.

   (f) Briefly explain the effects of charge injection and clock feed-through on the performance of this circuit.

   (g) Suggest two ways for reducing the effects of charge injection and clock feed-through in this circuit, and draw the schematic diagram of the modified circuit.

   ![Switch-capacitor circuit diagram]

Solution (2)
(a) Switched-capacitor based amplifier. Both $C_H$ and $C_F$ are initially charged to $v_{\text{in}} - v_{\text{offset}}$. Then charge in $C_H$ is transferred to $C_F$, amplifying $V_{\text{in}}$ while defining a new value for $V_{\text{out}}$.

(b) $\Phi_1$ 
(c) $V_{\text{out}} = V_{\text{in}} \left( 1 + \frac{R_2}{R_1} \right)$
(d) Advantages: (1) The amplifier can also cancel the offset voltage of the OpAmp. (2) No static power is needed in the feedback loop. Therefore, an OTA will be sufficient. (3) No thermal noise is generated from the capacitors.

Disadvantages: (1) High-frequency clock which should be much higher than Vin bandwidth is needed. (2) Switches induce KT/C noise. (3) Charge injection and clock feed-through.

(e) After $t_2$, both $C_H$ and $C_F$ are initially charged to $V_{in} - V_{offset}$: $Q_{total2} = C_H(V_{in} - V_{offset}) + C_F(V_{in} - V_{offset})$

After $t_3$, $V_{CH} = 0V - V_{offset}$, and $V_{CF} = V_{out} - V_{offset}$, and $Q_{total3} = C_H(0 - V_{offset}) + C_F(v_{out} - V_{offset})$

Due to conservation of charge and the fact that $C_H$ and $C_F$ are connected in series, $Q_{total2} = Q_{total3}$

Therefore: $v_{out} = v_{in} * (1 + C_H/C_F)$

(f) $\Phi_1$ charge injection: This is independent of $v_{in}$ because the negative terminal of the OpAmp is at GND or $V_{cm}$. Therefore, this charge only creates an offset which can be removed by the following stage or after digitization.

$\Phi_2$ charge injection: This is dependent on $v_{in}$, however, when $\Phi_2$ opens, the right size of both switches face high impedance, i.e. current cannot flow into $C_H$ and $C_F$, and therefore will return mostly back into the $v_{in}$ node.

$\Phi_3$ charge injection: This is dependent on $v_{in}$, however, when $\Phi_3$ opens, similar to $\Phi_2$, the current will have a difficult time passing through $C_H$ and $C_F$. Therefore, the current chooses to flow into the lower resistance path, i.e. the output node and the GND.

(g) 1. Add dummy switches. 2. Use a fully-differential amplifier structure:
3. In the following common-source amplifier find expressions for:

(a) Input referred noise voltage.
(b) Input referred noise current.
(c) Suggest three ways to reduce noise in this circuit.

Solution (3)

(a) \[ V_{o,\text{noise}}^2 = \left( \frac{4kT}{3g_{m1}} + \frac{K_x I_{D}^F}{f(C_{ox})^2W/L} + \frac{4kT}{R_D} \right) \times R_D^2 \]

\[ V_{i,\text{noise}}^2 = \frac{V_{o,\text{noise}}^2}{A_{V^2}} \approx \frac{V_{o,\text{noise}}^2}{(g_{m1}R_D)^2} = 4kT \cdot \frac{2}{3g_{m1}} + \frac{K_x I_{D}^F}{f(C_{ox})^2W/L g_{m1}^2} + \frac{4kT}{g_{m1}^2 R_D} \]

(b) \[ I_{i,\text{noise}}^2 = \frac{V_{i,\text{noise}}^2}{Z_{in}^2} = V_{i,\text{noise}}^2 \times (2\pi f \times C_{in})^2 \]

(c) 1. Increase \( g_{m1} \) (increase \( W/L1 \)) to reduce all noises according to \( V_{i,\text{noise}} \) expression.
2. Increase \( R_D \) to reduce thermal noise of \( R_D \).
3. Increase \( W/L \times L1 \) to reduce 1/f noise of \( M_1 \).