LECTURE 380 – TEMPERATURE INDEPENDENT REFERENCES
(READING: Text-Sec. 4.4.3)

INTRODUCTION

Objective
The objective of this presentation is:
1.) Introduce circuits that have various degrees of temperature independence
2.) Provide a background for the bandgap voltage references to follow

Outline
• Simple bias circuits
• Bootstrapped bias circuits
• Summary
Characterization of Temperature Dependence

The objective is to minimize the fractional temperature coefficient defined as,

\[ TCF = \frac{1}{V_{REF}} \left( \frac{\partial V_{REF}}{\partial T} \right) = \frac{1}{T} \frac{V_{REF}}{S} \text{ parts per million per } ^\circ C \text{ or ppm/} ^\circ C \]

Temperature dependence of PN junctions:

\[ i \approx I_s \exp\left( \frac{V}{V_t} \right) \]

\[ I_s = KT^3 \exp\left( \frac{-V_{GO}}{V_t} \right) \]

\[ \frac{dv_{BE}}{dT} \approx \frac{V_{BE} - V_{GO}}{T} = -2mV/ ^\circ C \text{ at room temperature} \]

\( (V_{GO} = 1.205 \text{ V at room temperature and is called the bandgap voltage}) \)

Temperature dependence of MOSFET in strong inversion:

\[ \frac{dv_{GS}}{dT} = \frac{dV_T}{dT} + \sqrt{\frac{2L}{WC_{ox}}} \frac{d}{dT} \left( \sqrt{\frac{i_D}{\mu_o}} \right) \]

\[ \mu_o = KT^{-1.5} \]

\[ V_T(T) = V_T(T_0) - \alpha(T - T_0) \]

Resistors:

\[ \frac{1}{R} \frac{dR}{dT} \text{ ppm/} ^\circ C \]
Bipolar-Resistance Voltage References

From previous work we know that,

\[ V_{REF} = \frac{kT}{q} \ln \left( \frac{V_{DD} - V_{REF}}{RI_s} \right) \]

However, not only is \( V_{REF} \) a function of \( T \), but \( R \) and \( I_s \) are also functions of \( T \).

\[ \therefore \quad \frac{dV_{REF}}{dT} = \frac{k}{q} \ln \left( \frac{V_{DD} - V_{REF}}{RI_s} \right) + \frac{kT}{q} \left( \frac{RI_s}{V_{DD} - V_{REF}} \right) - 1 \cdot \frac{dV_{REF}}{dT} - \left( \frac{V_{DD} - V_{REF}}{RI_s} \right) \left( \frac{dR}{RdT} + \frac{dI_s}{I_s dT} \right) \]

\[ = \frac{V_{REF}}{T} - \frac{V_t}{V_{DD} - V_{REF}} \frac{dV_{REF}}{dT} - V_t \left( \frac{dR}{RdT} + \frac{dI_s}{I_s dT} \right) = \frac{V_{REF} - V_{GO}}{T} - \frac{V_t}{V_{DD} - V_{REF}} \frac{dV_{REF}}{dT} - \frac{3V_t}{T} - \frac{V_t}{R} \frac{dR}{dT} \]

\[ \therefore \quad \frac{dV_{REF}}{dT} = \frac{V_{REF} - V_{GO}}{T} - \frac{V_t}{R} \frac{dR}{dT} - \frac{3V_t}{T} \approx \frac{V_{REF} - V_{GO}}{T} - \frac{V_t}{R} \frac{dR}{dT} - \frac{3V_t}{T} \]

\[ TC_F = \frac{1}{V_{REF}} \frac{dV_{REF}}{dT} = \frac{V_{REF} - V_{GO}}{V_{REF} \cdot T} - \frac{V_t}{V_{REF}} \frac{dR}{RdT} - \frac{3V_t}{V_{REF} \cdot T} \]

If \( V_{REF} = 0.6V \), \( V_t = 0.026V \), and the \( R \) is polysilicon, then at 27°C the \( TC_F \) is

\[ TC_F = \frac{0.6 - 1.205}{0.6} - \frac{0.026 \cdot 0.0015}{0.6} - \frac{3 \cdot 0.026}{0.6} = 33110^{-6} - 65 \times 10^{-6} - 433 \times 10^{-6} = -3859 \text{ ppm/°C} \]

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MOSFET Resistor Voltage Reference

From previous results we know that

$$V_{REF} = V_{GS} = V_T + \sqrt{\frac{2(V_{DD} - V_{REF})}{\beta R}}$$

or

$$V_{REF} = V_T - \frac{1}{\beta R} + \sqrt{\frac{2(V_{DD} - V_T)}{\beta R}} + \frac{1}{(\beta R)^2}$$

Note that $V_{REF}$, $V_T$, $\beta$, and $R$ are all functions of temperature. It can be shown that the $TC_F$ of this reference is

$$\frac{dV_{REF}}{dT} = -\alpha + \sqrt{\frac{V_{DD} - V_{REF}}{2\beta R}} \left( \frac{1.5}{T} - \frac{1}{R} \frac{dR}{dT} \right)$$

$$1 + \frac{1}{\sqrt{2\beta R (V_{DD} - V_{REF})}}$$

\[ V_{REF} \left(1 + \frac{1}{\sqrt{2\beta R (V_{DD} - V_{REF})}} \right) \]

\[ TC_F = \]
Example 1 - Calculation of MOSFET-Resistor Voltage Reference $TC_F$

Calculate the temperature coefficient of the MOSFET-Resistor voltage reference where W/L=2, $V_{DD}=5V$, $R=100k\Omega$ using the parameters of Table 3.1-2. The resistor, R, is polysilicon and has a tempco of 1500 ppm/°C.

**Solution**

First, calculate $V_{REF}$. Note that $\beta R = 220 \times 10^{-6} \times 10^5 = 22$ and $\frac{dR}{RdT} = 1500\text{ppm/°C}$

\[ \therefore V_{REF} = 0.7 - \frac{1}{22} + \sqrt{\frac{2(5 - 0.7)}{22}} + \left(\frac{1}{22}\right)^2 = 1.281\text{V} \]

Now,

\[ \frac{dV_{REF}}{dT} = \frac{-2.3 \times 10^{-3} + \sqrt{\frac{5 - 1.281}{2(22)}} \left(\frac{1.5}{300} - 1500 \times 10^{-6}\right) \left(1 + \frac{1}{\sqrt{2(22)(5 - 1.281)}} \right)} = -1.189 \times 10^{-3} \text{V/°C} \]

The fractional temperature coefficient is given by

\[ TCF = -1.189 \times 10^{-3} \left(\frac{1}{1.281}\right) = -928 \text{ ppm/°C} \]
Bootstrapped Current Source/Sink

Gate-source referenced source:

The output current was given as, \( I_{out} = \frac{V_{T1}}{R} + \frac{1}{\beta_1 R^2} + \frac{1}{R} \sqrt{\frac{2V_{T1}}{\beta_1 R} + \frac{1}{(\beta_1 R)^2}} \)

Although we could grind out the derivative of \( I_{out} \) with respect to \( T \), the temperature performance of this circuit is not that good to spend the time to do so. Therefore, let us assume that \( V_{GS1} \approx V_{T1} \) which gives

\[ I_{out} \approx \frac{V_{T1}}{R} \quad \Rightarrow \quad \frac{dI_{out}}{dT} = \frac{1}{R} \frac{dV_{T1}}{dT} - \frac{1}{R^2} \frac{dR}{dT} \]

In the resistor is polysilicon, then

\[ TC_F = \frac{1}{I_{out}} \frac{dI_{out}}{dT} = \frac{1}{V_{T1}} \frac{dV_{T1}}{dT} - \frac{1}{R} \frac{dR}{dT} = \frac{-\alpha}{V_{T1}} - \frac{1}{R} \frac{dR}{dT} = \frac{-2.3 \times 10^{-3}}{0.7} - 1.5 \times 10^{-3} = -4786 \text{ppm/}^\circ\text{C} \]

Base-emitter referenced source:

The output current was given as, \( I_{out} = I_2 = \frac{V_{BE1}}{R} \)

The \( TC_F = \frac{1}{V_{BE1}} \frac{dV_{BE1}}{dT} - \frac{1}{R} \frac{dR}{dT} \)

If \( V_{BE1} = 0.6 \text{V} \) and \( R \) is poly, then the \( TC_F = \frac{1}{0.6} (-2 \times 10^{-3}) - 1.5 \times 10^{-3} = -4833 \text{ppm/}^\circ\text{C} \).
Zero Temperature Coefficient (ZTC) Point for MOSFETs

For a given value of gate-source voltage, it will be shown that the drain current of the MOSFET will be independent of temperature.

Consider the following circuit:

Assume that the transistor is saturated and that:

\[ \mu = \mu_o \left(\frac{T}{T_0}\right)^{-1.5} \]

and

\[ V_T(T) = V_T(T_0) + \alpha(T-T_0) \]

where \( \alpha = -0.0023 \text{V/°C} \) and \( T_0 = 27 \text{°C} \)

:: \( I_D(T) = \frac{\mu_o C_{ox} W}{2L} \left(\frac{T}{T_0}\right)^{-1.5} [V_{GS} - V_{T0} - \alpha(T-T_0)]^2 \)

\[ \frac{dI_D}{dT} = \frac{-1.5 \mu_o C_{ox} T}{2T_0} \left[\left(\frac{T}{T_0}\right)^{-2.5} [V_{GS} - V_{T0} - \alpha(T-T_0)]^2 + \alpha \mu_o C_{ox} \left(\frac{T}{T_0}\right)^{-1.5} [V_{GS} - V_{T0} - \alpha(T-T_0)] \right] = 0 \]

:: \( V_{GS} - V_{T0} - \alpha(T-T_0) = \frac{-4T\alpha}{3} \)

\[ \Rightarrow V_{GS}(ZTC) = V_{T0} - \alpha T_0 - \frac{\alpha T}{3} \]

Let \( K' = 10 \mu A/V^2 \), \( W/L = 5 \) and \( V_{T0} = 0.71 \text{V} \).

At \( T = 27°C(300°K) \), \( V_{GS}(ZTC) = 0.71 - (-23 \text{mV})(300°K) - (0.333)(-23 \text{mV})(300°K) = 1.63 \text{V} \)

At \( T = 27°C (300°K) \), \( I_D = (10 \mu A/V^2)(5/2)(1.63-0.71)^2 = 21.2 \mu A \)

At \( T = 200°C(473°K) \), \( V_{GS}(ZTC) = 0.71 - (-23 \text{mV})(300°K) - (0.333)(-23 \text{mV})(473°K) = 1.76 \text{V} \)
Experimental Verification of the ZTC Point

The data below is for a 5µm n-channel MOSFET with

W/L=50µm/10µm, \( N_A = 10^{-16} \text{cm}^{-3} \), \( t_{ox} = 650\text{Å} \), \( \mu_o C_{ox} = 10\mu\text{A/V}^2 \), and \( V_{T0} = 0.71\text{V} \).
ZTC Point for PMOS

The data below is for a 5µm p-channel MOSFET with

\[ W/L = 50\mu m/10\mu m, \quad N_D = 2 \times 10^{-15} \text{cm}^{-3}, \quad \text{and} \quad t_{ox} = 650\text{Å}. \]

Zero temperature coefficient will occur for every MOSFET up to about 200°C.
**Technique to Make $g_m$ Dependent on a Resistor**

Consider the following circuit with all transistor having a $W/L = 10$. This is a bootstrapped reference which creates a $V_{\text{bias}}$ independent of $V_{DD}$. The two key equations are:

\[ I_3 = I_4 \Rightarrow I_1 = I_2 \]

and

\[ V_{GS1} = V_{GS2} + I_2R \]

Solving for $I_2$ gives:

\[ I_2 = \frac{V_{GS1} - V_{GS2}}{R} = \frac{1}{R} \left( \sqrt{\frac{2I_1}{\beta_1}} - \sqrt{\frac{2I_2}{\beta_2}} \right) = \frac{\sqrt{2I_1}}{R\sqrt{\beta_1}} \left( 1 - \frac{1}{2} \right) \]

\[ \therefore \sqrt{I_2} = \frac{1}{R\sqrt{2\beta_1}} \Rightarrow I_2 = I_1 = \frac{1}{2\beta_1R^2} = \frac{1}{2.24 \times 10^{-6} \cdot 10^8} = 20.833 \mu A \]

Now, $V_{\text{bias}}$ can be written as

\[ V_{\text{bias}} = V_{GS1} = \sqrt{\frac{2I_2}{\beta_1}} + V_{TN} = \frac{1}{\beta_1R} + V_{TN} = \frac{1}{110 \times 10^{-6} \cdot 10^{-4}} + 0.7 = 0.0909 + 0.7 = 0.7909 V \]

Any transistor with $V_{GS} = V_{\text{bias}}$ will have a current flow that is given by $1/2\beta R^2$.

Therefore, $g_m = \sqrt{2I_\beta} = \sqrt{\frac{2\beta}{2\beta R^2}} = \frac{1}{R} \Rightarrow g_m = \frac{1}{R}$

(This means that the temperature dependence of $g_m$ will be that of $1/R$ which can be used to achieve temperature controlled performance.)
### SUMMARY

<table>
<thead>
<tr>
<th>Type of Reference</th>
<th>$S_{V_{REF}/V_{DD}}$</th>
<th>$TC_F$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOSFET-R</td>
<td>&lt;1</td>
<td>&gt;1000ppm/°C</td>
<td></td>
</tr>
<tr>
<td>BJT-R</td>
<td>&lt;&lt;1</td>
<td>&gt;1000ppm/°C</td>
<td></td>
</tr>
<tr>
<td>Breakdown Diode</td>
<td>&lt;&lt;1</td>
<td>Can be very small</td>
<td>BV too large</td>
</tr>
<tr>
<td>Bootstrap Gate-Source Referenced</td>
<td>Good if currents are matched</td>
<td>&gt;1000ppm/°C</td>
<td>Requires start-up circuit</td>
</tr>
<tr>
<td>Bootstrap Base-emitter Referenced</td>
<td>Good if currents are matched</td>
<td>&gt;1000ppm/°C</td>
<td>Requires start-up circuit</td>
</tr>
</tbody>
</table>

- A MOSFET can have zero temperature dependence of $i_D$ for a certain $v_{GS}$
- If one is careful, very good independence of power supply can be achieved
- None of the above references have really good temperature independence

Consider the following example:

A 10 bit ADC has a reference voltage of 1V. The LSB is approximately 0.001V. Therefore, the voltage reference must be stable to within 0.1%. If a 100°C change in temperature is experienced, then the $TC_F$ must be 0.001%/C or multiplying by $10^4$ gives a $TC_F = 10$ppm/°C.