LECTURE 430 – COMPENSATION OF OP AMPS-II
(READING: Text-Sec. 9.2, 9.3, 9.4)

INTRODUCTION
The objective of this presentation is to continue the ideas of the last lecture on compensation of op amps.

Outline
• Compensation of Op Amps
  General principles
  Miller, Nulling Miller
  Self-compensation
  Feedforward
• Summary
Conditions for Stability of the Two-Stage Op Amp (Assuming $p_3 \geq GB$

• Unity-gain bandwidth is given as:

$$GB = A_v(0) \cdot |p_1| = \frac{g_m I \cdot R_I R_{II}}{g_m I I R_{II} C_c} = \frac{g_m I}{C_c} = \frac{g_m I g_m I I R_{I} R_{II}}{g_m I I R_{II} C_c} = \frac{g_m I}{C_c}$$

• The requirement for $45^\circ$ phase margin is:

$$\pm 180^\circ - \arg[AF] = \pm 180^\circ - \tan^{-1}\left(\frac{\omega}{|p_1|}\right) - \tan^{-1}\left(\frac{\omega}{|p_2|}\right) - \tan^{-1}\left(\frac{\omega}{z}\right) = 45^\circ$$

Let $\omega = GB$ and assume that $z \geq 10GB$, therefore we get,

$$\pm 180^\circ - \tan^{-1}\left(\frac{GB}{|p_1|}\right) - \tan^{-1}\left(\frac{GB}{|p_2|}\right) - \tan^{-1}\left(\frac{GB}{z}\right) = 45^\circ$$

$$135^\circ \approx \tan^{-1}(A_v(0)) + \tan^{-1}\left(\frac{GB}{|p_2|}\right) + \tan^{-1}(0.1) = 90^\circ + \tan^{-1}\left(\frac{GB}{|p_2|}\right) + 5.7^\circ$$

$$39.3^\circ \approx \tan^{-1}\left(\frac{GB}{|p_2|}\right) \Rightarrow \frac{GB}{|p_2|} = 0.818 \Rightarrow |p_2| \geq 1.22GB$$

• The requirement for $60^\circ$ phase margin:

$$|p_2| \geq 2.2GB \text{ if } z \geq 10GB$$

• If $60^\circ$ phase margin is required, then the following relationships apply:

$$\frac{g_m \cdot 6}{C_c} > \frac{10g_m I}{C_c} \Rightarrow g_m \cdot 6 > 10g_m I \text{ and } \frac{g_m \cdot 6}{C_2} > \frac{2.2g_m I}{C_c} \Rightarrow C_c > 0.22C_2$$
Controlling the Right-Half Plane Zero

Why is the RHP zero a problem?
Because it boosts the magnitude but lags the phase - the worst possible combination for stability.

Solution of the problem:
If zeros are caused by two paths to the output, then eliminate one of the paths.
Use of Buffer to Eliminate the Feedforward Path through the Miller Capacitor

Model:

The transfer function is given by the following equation,

\[
\frac{V_o(s)}{V_{in}(s)} = \frac{(g_{mi})(g_{mII})(R_I)(R_{II})}{1 + s[R_IC_I + R_{II}C_{II} + R_IC_c + g_{mII}R_IR_{II}C_c] + s^2[R_IR_{II}C_{II}(C_I + C_c)]}
\]

Using the technique as before to approximate \(p_1\) and \(p_2\) results in the following

\[p_1 \approx -\frac{-1}{R_IC_I + R_{II}C_{II} + R_IC_c + g_{mII}R_IR_{II}C_c} \approx -\frac{-1}{g_{mII}R_IR_{II}C_c}\]

and

\[p_2 \approx \frac{-g_{mII}C_c}{C_{II}(C_I + C_c)}\]

Comments:

Poles are approximately what they were before with the zero removed.

For 45° phase margin, \(|p_2|\) must be greater than \(GB\)

For 60° phase margin, \(|p_2|\) must be greater than 1.73\(GB\)
Use of Buffer with Finite Output Resistance to Eliminate the RHP Zero

Assume that the unity-gain buffer has an output resistance of $R_O$.

Model:

It can be shown that if the output resistance of the buffer amplifier, $R_O$, is not neglected that another pole occurs at

$$p_4 \cong -\frac{1}{R_O[C_I C_c/(C_I + C_c)]}$$

and a LHP zero at

$$z_2 \cong -\frac{1}{R_O C_c}$$

Closer examination shows that if a resistor, called a nulling resistor, is placed in series with $C_c$ that the RHP zero can be eliminated or moved to the LHP.
Use of Nulling Resistor to Eliminate the RHP Zero (or turn it into a LHP zero)†

Nodal equations:

\[ g_{mI}V_{in} + \frac{V_I}{R_I} + sC_I V_I + \left( \frac{sC_c}{1 + sC_c R_Z} \right) (V_I - V_{out}) = 0 \]

\[ g_{mII}V_I + \frac{V_o}{R_{II}} + sC_{II} V_{out} + \left( \frac{sC_c}{1 + sC_c R_Z} \right) (V_{out} - V_I) = 0 \]

Solution:

\[ \frac{V_{out}(s)}{V_{in}(s)} = \frac{a \{1 - s[(C_c/g_{mII}) - R_Z C_c]\}}{1 + b s + c s^2 + d s^3} \]

where

\[ a = g_{mI}g_{mII} R_I R_{II} \]
\[ b = (C_{II} + C_c) R_{II} + (C_I + C_c) R_I + g_{mII} R_I R_{II} C_c + R_Z C_c \]
\[ c = [R_I R_{II} (C_I C_{II} + C_c C_I + C_c C_{II}) + R_Z C_c (R_I C_I + R_{II} C_{II})] \]
\[ d = R_I R_{II} R_Z C_I C_{II} C_c \]

Use of Nulling Resistor to Eliminate the RHP - Continued

If $R_z$ is assumed to be less than $R_I$ or $R_{II}$ and the poles widely spaced, then the roots of the above transfer function can be approximated as

\[ p_1 \approx \frac{-1}{(1 + g_{mII}R_{II})R_I C_c} \approx \frac{-1}{g_{mII}R_{II}R_I C_c} \]

\[ p_2 \approx \frac{-g_{mII}C_c}{C_I C_{II} + C_c C_I + C_c C_{II}} \approx \frac{-g_{mII}}{C_{II}} \]

\[ p_4 = \frac{-1}{R_z C_I} \]

and

\[ z_1 = \frac{1}{C_c(1/g_{mII} - R_z)} \]

Note that the zero can be placed anywhere on the real axis.
Conceptual Illustration of the Nulling Resistor Approach

The output voltage, $V_{out}$, can be written as

$$V_{out} = \frac{-g_{m6} R_{II} \left( R_{z} + \frac{1}{sC_c} \right)}{R_{II} + R_{z} + \frac{1}{sC_c}} V' + \frac{R_{II}}{R_{II} + R_{z} + \frac{1}{sC_c}} V'' = \frac{-R_{II} \left[ g_{m6} R_{z} + \frac{g_{m6}}{sC_c} - 1 \right]}{R_{II} + R_{z} + \frac{1}{sC_c}} V$$

when $V = V' = V''$.

Setting the numerator equal to zero and assuming $g_{m6} = g_{mII}$ gives,

$$z_1 = \frac{1}{C_c \left( 1/g_{mII} - R_{z} \right)}$$
A Design Procedure that Allows the RHP Zero to Cancel the Output Pole, \( p_2 \)

We desire that \( z_1 = p_2 \) in terms of the previous notation. Therefore,

\[
\frac{1}{C_c(1/g_{mII} - R_z)} = \frac{-g_{mII}}{C_{II}}
\]

The value of \( R_z \) can be found as

\[
R_z = \left( \frac{C_c + C_{II}}{C_c} \right) (1/g_{mII})
\]

With \( p_2 \) canceled, the remaining roots are \( p_1 \) and \( p_4 \) (the pole due to \( R_z \)). For unity-gain stability, all that is required is that

\[
|p_4| > A_v(0)|p_1| = \frac{A_v(0)}{g_{mII}R_{II}R_I C_c} = \frac{g_{mI}}{C_c}
\]

and

\[
\left( \frac{1}{R_z C_I} \right) > \left( \frac{g_{mI}}{C_c} \right) = GB
\]

Substituting \( R_z \) into the above inequality and assuming \( C_{II} >> C_c \) results in

\[
C_c > \sqrt{\frac{g_{mI}}{g_{mII} C_I C_{II}}}
\]

This procedure gives excellent stability for a fixed value of \( C_{II} (\approx C_L) \). Unfortunately, as \( C_L \) changes, \( p_2 \) changes and the zero must be readjusted to cancel \( p_2 \).
Increasing the Magnitude of the Output Pole†

The magnitude of the output pole, \( p_2 \), can be increased by introducing gain in the Miller capacitor feedback path. For example,

![电路图](image)

The resistors \( R_1 \) and \( R_2 \) are defined as

\[
R_1 = \frac{1}{g_{ds2} + g_{ds4} + g_{ds9}} \quad \text{and} \quad R_2 = \frac{1}{g_{ds6} + g_{ds7}}
\]

where transistors M2 and M4 are the output transistors of the first stage.

Nodal equations:

\[
I_{in} = G_1 V_1 - g_{m8} V_{s8} = G_1 V_1 \left( \frac{g_{m8} s C_c}{g_{m8} + s C_c} \right) V_{out} \quad \text{and} \quad 0 = g_{m6} V_1 + \left[ G_2 + s C_2 + \frac{g_{m8} s C_c}{g_{m8} + s C_c} \right] V_{out}
\]

Increasing the Magnitude of the Output Pole - Continued

Solving for the transfer function \( \frac{V_{out}}{I_{in}} \) gives,

\[
\frac{V_{out}}{I_{in}} = \left( \frac{-g_{m6}}{G_1 G_2} \right) \left[ \frac{1 + \frac{sC_c}{g_{m8}}}{1 + s \left[ \frac{C_c}{g_{m8}} + \frac{C_2}{G_2} + \frac{C_c}{G_2} + \frac{g_{m6}C_c}{G_1 G_2} \right] + s^2 \left( \frac{C_c C_2}{g_{m8} G_2} \right)} \right]
\]

Using the approximate method of solving for the roots of the denominator gives

\[
p_1 = \left[ \frac{1}{C_c G_2 + \frac{2}{g_{m6}} - \frac{1 - 6}{g_{m6}^2 C_c r_{ds}}} \right] \approx \frac{-6}{g_{m6} r_{ds}^2 C_c}
\]

and

\[
p_2 \approx \left[ \frac{2}{C_c C_2 - \frac{6}{g_{m8} G_2}} \right] \frac{g_{m8} r_{ds}^2 G_2}{6} \left( \frac{g_{m6}}{C_2} \right) = \left( \frac{g_{m8} r_{ds}}{3} \right) |p_2'|
\]

where all the various channel resistance have been assumed to equal \( r_{ds} \) and \( p_2' \) is the output pole for normal Miller compensation.

Result:

Dominant pole is approximately the same and the output pole is increased by \( \approx g_{m} r_{ds} \).
Concept Behind the Increasing of the Magnitude of the Output Pole

\[ R_{out} = r_{ds7} \left| \frac{3}{g_{m6}g_{m8}r_{ds8}} \right| \approx \frac{3}{g_{m6}g_{m8}r_{ds8}} \]

Therefore, the output pole is approximately,

\[ |p_2| \approx \frac{g_{m6}g_{m8}r_{ds8}}{3C_\Pi} \]
FEEDFORWARD COMPENSATION

Use two parallel paths to achieve a LHP zero for lead compensation purposes.

\[ \frac{V_{out}(s)}{V_{in}(s)} = \frac{AC_c}{C_c + C_{II}} \left( \frac{s + g_{mII}/AC_c}{s + 1/[R_{II}(C_c + C_{II})]} \right) \]

To use the LHP zero for compensation, a compromise must be observed.

- Placing the zero below GB will lead to boosting of the loop gain that could deteriorate the phase margin.
- Placing the zero above GB will have less influence on the leading phase caused by the zero.

Note that a source follower is a good candidate for the use of feedforward compensation.
SELF-COMPENSATED OP AMPS

Self compensation occurs when the load capacitor is the compensation capacitor (can never be unstable for resistive feedback)

Voltage gain:
\[ \frac{v_{out}}{v_{in}} = A_v(0) = G_m R_{out} \]

Dominant pole:
\[ p_1 = \frac{-1}{R_{out}C_L} \]

Unity-gainbandwidth:
\[ GB = A_v(0) \cdot |p_1| = \frac{G_m}{C_L} \]

Stability:
Large load capacitors simply reduce \( GB \) but the phase is still 90° at \( GB \).
SUMMARY

Compensation

• Designed so that the op amp with unity gain feedback (buffer) is stable

• Types
  - Miller
  - Miller with nulling resistors
  - Self Compensating
  - Feedforward