LECTURE 390 – BANDGAP VOLTAGE REFERENCE
(READING: Text-Sec. 4.4.3)

INTRODUCTION
The objective of this presentation is:
1.) Introduce the concept of a bandgap reference
2.) Show circuits that implement the bandgap reference

Outline
• Introduction
• Development of the bandgap circuit
• Bandgap reference circuits
• Summary
Temperature Stable References

- The previous reference circuits failed to provide small values of temperature coefficient although sufficient power supply independence was achieved.
- This lecture introduces the bandgap voltage concept combined with power supply independence to create a very stable voltage reference in regard to both temperature and power supply variations.

Bandgap Voltage Reference Principle

The principle of the bandgap voltage reference is to balance the negative temperature coefficient of a pn junction with the positive temperature coefficient of the thermal voltage, $V_t = kT/q$.

Concept:

Result: References with $TC_F$’s approaching 10 ppm/°C.
DEVELOPMENT OF THE BANDGAP REFERENCE CIRCUIT

Derivation of the Temperature Coefficient of the Base-Emitter Voltage

For small TC_F's the dependence V_{BE} must be known more precisely than \( \approx -2 \text{mV/}^\circ\text{C} \).

1.) Start with the collector current density, \( J_C \):

\[
J_C = \frac{q D_n n_{po}}{W_B} \exp \left( \frac{V_{BE}}{V_t} \right)
\]

where, \( J_C = \frac{I_C}{\text{Area}} = \text{collector current density} \)

\( D_n = \text{average diffusion constant for electrons} \)

\( W_B = \text{base width} \)

\( V_{BE} = \text{base-emitter voltage} \)

\( V_t = kT/q \)

\( k = \text{Boltzmann's constant} \ (1.38 \times 10^{-23} \text{J/}^\circ\text{K}) \)

\( T = \text{Absolute temperature} \)

\( n_{po} = \frac{n_i^2}{N_A} = \text{equilibrium concentration of electrons in the base} \)

\( n_i^2 = D T^3 \exp \left( \frac{-V_{GO}}{V_t} \right) = \text{intrinsic concentration of carriers} \)

\( D = \text{temperature independent constant} \)

\( V_{GO} = \text{bandgap voltage of silicon} \ (1.205\text{V}) \)

\( N_A = \text{acceptor impurity concentration} \)
Derivation of the Temperature Coefficient of the Base-Emitter Voltage - Continued

2.) Combine the above relationships into one:

\[ J_C = \frac{q D_n}{N_A W_B} DT^3 \exp\left(\frac{V_{BE} - V_{GO}}{V_t}\right) = AT^\gamma \exp\left(\frac{V_{BE} - V_{GO}}{V_t}\right) \]

where, \( \gamma = 3 \)

3.) The value of \( J_C \) at a reference temperature of \( T = T_0 \) is

\[ J_{C0} = AT_0^\gamma \exp\left[\frac{q}{kT_0} (V_{BE} - V_{GO})\right] \]

while the value of \( J_C \) at a general temperature, \( T \), is

\[ J_C = AT^\gamma \exp\left[\frac{q}{kT} (V_{BE} - V_{GO})\right] \]

4.) The ratio of \( J_C / J_{C0} \) can be expressed as,

\[ \frac{J_C}{J_{C0}} = \left(\frac{T}{T_0}\right)^\gamma \exp\left[\frac{q}{k} \left(\frac{V_{BE} - V_{GO}}{T} - \frac{V_{BE0} - V_{GO}}{T_0}\right)\right] \]

or

\[ \ln\left(\frac{J_C}{J_{C0}}\right) = \gamma \ln\left(\frac{T}{T_0}\right) + \frac{q}{kT} \left[ V_{BE} - V_{GO} - \frac{T}{T_0} (V_{BE0} - V_{GO}) \right] \]

where \( V_{BE0} \) is the value of \( V_{BE} \) at \( T = T_0 \).

5.) Solving for \( V_{BE} \) from the above results gives,

\[ V_{BE}(T) = V_{GO} \left(1 - \frac{T}{T_0}\right) + V_{BE0} \left(\frac{T}{T_0}\right) + \frac{\gamma kT}{q} \ln\left(\frac{T_0}{T}\right) + \frac{kT}{q} \ln\left(\frac{J_C}{J_{C0}}\right) \]
Derivation of the Temperature Coefficient of the Base-Emitter Voltage - Continued

6.) Next, assume $J_C \propto T^\alpha$ and find $\partial V_{BE}/\partial T$.

$$\frac{\partial V_{BE}}{\partial T} = \frac{\partial V_{GO}}{\partial T} \left( 1 - \frac{T}{T_0} \right) \frac{V_{GO}}{T_0} + \frac{V_{BE0}}{T_0} \frac{\gamma kT}{q} \frac{\partial \ln(T_0/T)}{\partial T} + \ln \left( \frac{T_0}{T} \right) \frac{\partial (\gamma kT/q)}{\partial T} + \frac{kT}{q} \left( \frac{\partial \ln(J_C/J_{C0})}{\partial T} \right) + q \ln \left( \frac{J_C}{J_{C0}} \right)$$

7.) Assume that $T = T_0$ which means $J_C = J_{C0}$. Since, $\partial V_{GO}/\partial T = 0$,

$$\left. \frac{\partial V_{BE}}{\partial T} \right|_{T=T_0} = - \frac{V_{GO}}{T_0} + \frac{V_{BE0}}{T_0} \frac{\gamma kT}{q} \frac{\partial \ln(T_0/T)}{\partial T} + \frac{kT}{q} \left( \frac{\partial \ln(J_C/J_{C0})}{\partial T} \right)$$

8.) Note that,

$$\frac{\partial \ln(T_0/T)}{\partial T} = \frac{T}{T_0} \frac{\partial (T_0/T)}{\partial T} = \frac{T}{T_0} \left( -\frac{T_0}{T^2} \right) = -1 \frac{T}{T} \quad \text{and} \quad \frac{\partial \ln(J_C/J_{C0})}{\partial T} = \frac{J_{C0}}{J_C} \frac{\partial (J_C/J_{C0})}{\partial T} = \frac{J_{C0}}{J_C} \left( \frac{\alpha J_C}{J_{C0}} \right) = \alpha$$

Therefore,

$$\left. \frac{\partial V_{BE}}{\partial T} \right|_{T=T_0} = - \frac{V_{GO}}{T_0} + \frac{V_{BE0}}{T_0} \frac{\gamma k}{q} + \frac{\alpha k}{q} \quad \text{or} \quad \left. \frac{\partial V_{BE}}{\partial T} \right|_{T=T_0} = \frac{V_{BE0} - V_{GO}}{T_0} + (\alpha - \gamma) \frac{k}{q}$$

Typical values of $\alpha$ and $\gamma$ are 1 and 3.2. If $V_{BE0} = 0.6V$, then at room temperature:

$$\left. \frac{\partial V_{BE}}{\partial T} \right|_{T=T_0} = \frac{0.6 - 1.205}{300} + (1 - 3.2) \left( \frac{0.026}{300} \right) = \frac{0.6 - 1.205 - 0.1092}{300} = -1.826 \text{mV/}^\circ \text{C}$$
Derivation of the Temperature Coefficient of the Thermal Voltage \((kT/q)\)

1.) Consider two identical pn junctions having different current densities,

\[ \Delta V_{BE} = V_{BE1} - V_{BE2} = kT/q \ln \left( \frac{J_{C1}}{J_{C2}} \right) \]

2.) Find \(\frac{\partial (\Delta V_{BE})}{\partial T}\),

\[ \frac{\partial (\Delta V_{BE})}{\partial T} = \frac{V_t}{T} \ln \left( \frac{J_{C1}}{J_{C2}} \right) = \frac{k}{q} \ln \left( \frac{J_{C1}}{J_{C2}} \right) \]
Derivation of the Gain, K, for the Bandgap Voltage Reference

1.) In order to achieve a zero temperature coefficient at $T = T_0$, the following equation must be satisfied:

$$0 = \left. \frac{\partial V_{BE}}{\partial T} \right|_{T=T_0} + K'' \frac{\partial(\Delta V_{BE})}{\partial T}$$

where $K''$ is a constant that satisfies the equation.

2.) Therefore, we get

$$0 = K'' \left( \frac{V_{t0}}{T_0} \right) \ln \left( \frac{J_{C1}}{J_{C2}} \right) + \frac{V_{BE0} - V_{GO}}{T_0} + \frac{(\alpha - \gamma)V_{t0}}{T_0}$$

3.) Define $K = K'' \ln \left( \frac{J_{C1}}{J_{C2}} \right)$, therefore

$$0 = K \left( \frac{V_{t0}}{T_0} \right) + \frac{V_{BE0} - V_{GO}}{T_0} + \frac{(\alpha - \gamma)V_{t0}}{T_0}$$

4.) Solving for $K$ gives

$$K = \frac{V_{GO} - V_{BE0} - V_{t0}(\alpha - \gamma)}{V_{t0}}$$

Assuming that $J_{C1}/J_{C2} = A_{E1}/A_{E2} = 10$ and $V_{BE0} = 0.6V$ gives,

$$K = \frac{1.205 - 0.6 + (2.2)(0.026)}{0.026} = 25.469$$

5.) The output voltage of the bandgap voltage reference is found as,

$$V_{REF|T=T_0} = V_{BE0} + KV_{t0} = V_{BE0} + V_{GO} - V_{BE0} + (\gamma - \alpha)V_{t0} \quad \text{or} \quad V_{REF} = V_{GO} + (\gamma - \alpha)V_{t0}$$

For the previous values, $V_{REF} = 1.205 + 0.026(2.2) = 1.262V$. 

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Variation of the Bandgap Reference Voltage with respect to Temperature

The previous derivation is only valid at a given temperature, $T_0$. As the temperature changes away from $T_0$, the value of $\frac{\partial V_{\text{REF}}}{\partial T}$ is no longer zero.

Illustration:

![Graph showing variation of the Bandgap Reference Voltage with respect to Temperature](image)

Bandgap curvature correction will be necessary for low ppm/C bandgap references.
BANDGAP REFERENCE CIRCUITS

Classical Widlar Bandgap Voltage Reference

Operation:

\[ V_{BE1} = V_{BE2} + I_2 R_3 \]

gives

\[ \Delta V_{BE} = V_{BE1} - V_{BE2} = I_2 R_3 \]

But,

\[ \Delta V_{BE} = V_t \ln \left( \frac{I_1}{I_{s1}} \right) - V_t \ln \left( \frac{I_2}{I_{s2}} \right) = V_t \ln \left( \frac{I_1 I_{s2}}{I_2 I_{s1}} \right) \]

Assume \( V_{BE1} \approx V_{BE3} \), we get \( I_1 R_1 = I_2 R_2 \)

Therefore,

\[ I_2 = \frac{\Delta V_{BE}}{R_3} = \frac{V_t}{R_3} \ln \left( \frac{I_1 I_{s2}}{I_2 I_{s1}} \right) = \frac{V_t}{R_3} \ln \left( \frac{R_2 I_{s2}}{R_1 I_{s1}} \right) \]

Now we can express \( V_{REF} \) as

\[ V_{REF} = I_2 R_2 + V_{BE3} = \frac{R_2}{R_3} V_t \ln \left( \frac{R_2 I_{s2}}{R_1 I_{s1}} \right) + V_{BE3} = KV_t + V_{BE} \]

Design \( R_1, R_2, I_{s1}, \) and \( I_{s2} \) to get the desired \( K \).

Let \( K = 25 \) and \( I_{s2} = 10 I_{s1} \) and design \( R_1, R_2, \) and \( R_3 \). Choose \( R_2 = 10 R_1 = 10k\Omega \).

Therefore, \( \ln(100) = 4.602 \). Therefore \( R_2/R_3 = 25/4.602 \) or \( R_3 = R_2/5.4287 = 1.842k\Omega \).

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A CMOS Bandgap Reference using PNP Lateral BJTs

Bootstrapped Voltage Reference using PNP Laterals-

\[ I_2 = \frac{V_{BE1} - V_{BE2}}{R_2} = \frac{V_t}{R_2} \left[ \ln \left( \frac{I_1}{I_{s1}} \right) - \ln \left( \frac{I_2}{I_{s2}} \right) \right] = \frac{V_t}{R_2} \ln \left( \frac{I_{s2}}{I_{s1}} \right) = \frac{V_t}{R_2} \ln \left( \frac{A_{E2}}{A_{E1}} \right) \]

if \( I_1 = I_2 \) which is forced by the current mirror consisting of M1 and M2.

\[ V_{REF} = V_{BE1} + I_1R_1 = V_{BE1} + \left( \frac{R_1}{R_2} \ln \left( \frac{A_{E2}}{A_{E1}} \right) \right) V_t = V_{BE1} + KV_t \]

While an op amp could be used to make \( I_1 = I_2 \) it suffers from offset and noise and leads to deterioration of the bandgap temperature performance.

\( V_{REF} \) is with respect to \( V_{DD} \) and therefore is susceptible to changes on \( V_{DD} \).
A CMOS Bandgap Reference using Substrate PNP BJTs

Operation:

The cascode mirror (M5-M8) keeps the currents in Q1, Q2, and Q3 identical. Thus,
\[ V_{BE1} = I_2 R + V_{BE2} \]
or
\[ I_2 = \frac{V_t}{R} \ln(n) \]

Therefore,
\[ V_{REF} = V_{BE3} + I_2(kR) = V_{BE3} + kV_t \cdot \ln(n) \]
Use \( k \) and \( n \) to design the desired value of \( K \) (\( n \) is an integer greater than 1).
Weak Inversion Bandgap Voltage Reference
Circuit:

Analysis:

For the p-channel transistors:

\[ I_D = I_{DO}(W/L) \exp\left(\frac{V_{BG}}{nV_t}\right) \exp\left(-\frac{V_{BS}}{V_t}\right) - \exp\left(-\frac{V_{BD}}{V_t}\right) \]

where \( V_t = kT/q \).

If \( V_{BD} \gg V_t \), then \( I_D = I_{DO}(W/L) \exp\left(\frac{V_{BG}}{nV_t} - \frac{V_{BS}}{V_t}\right) \).

The various transistor currents can be expressed as:

\[ I_{D1} = I_{D2} = I_{DO}(W_2/L_2) \exp\left(\frac{V_{BG2}}{nV_t}\right) \quad \text{and} \quad I_{D3} = I_{D4} = I_{DO}(W_4/L_4) \exp\left(\frac{V_{BG4}}{nV_t} - \frac{V_{BS4}}{V_t}\right) \]

Note that \( V_{BG2} = V_{BG4} \) and \( V_{BS4} = V_{R1} \).

Therefore,

\[ \frac{I_{D1}}{I_{D3}} = \frac{W_2/L_2}{W_4/L_4} \exp\left(\frac{V_{R1}}{V_t}\right) \]

where

\[ V_{R1} = V_t \ln\left(\frac{W_1 W_4 L_2 L_3}{L_1 L_4 W_2 W_3}\right) \quad \text{and} \quad I_{R1} = \frac{V_{R1}}{R_1} \]
Weak Inversion Bandgap Voltage Reference - Continued

The reference voltage can be expressed as,

\[ V_{\text{REF}} = R_2 I_6 + V_{\text{BE5}} \]

However,

\[ I_6 = \frac{W_6 L_3}{L_6 W_3} \quad I_{R1} = \frac{W_6 L_3}{L_6 W_3} \frac{V_t}{R_1} \ln \left( \frac{W_1 W_4 L_2 L_3}{L_1 L_4 W_2 W_3} \right) . \]

Substituting \( I_6 \) and the previously derived expression for \( V_{\text{BE}}(T) \) in \( V_{\text{REF}} \) gives,

\[ V_{\text{REF}} = \frac{W_6 L_3}{L_6 W_3} \frac{R_2}{R_1} V_t \ln \left( \frac{W_1 W_4 L_2 L_3}{L_1 L_4 W_2 W_3} \right) + V_{\text{GO}} \left( 1 - \frac{T}{T_0} \right) + V_{\text{BE0}} \left( \frac{T}{T_0} \right) + 3V_t \ln \left( \frac{T_0}{T} \right) \]

To achieve \( \partial V_{\text{REF}} / \partial T = 0 \) at \( T = T_0 \), we get

\[ \frac{\partial V_{\text{REF}}}{\partial T} = \left( \frac{k}{q} \right) \frac{R_2}{R_1} \frac{W_6 L_3}{L_6 W_3} \ln \left( \frac{W_1 W_4 L_2 L_3}{L_1 L_4 W_2 W_3} \right) - \frac{V_{\text{GO}}}{T_0} + \frac{V_{\text{BE0}}}{T_0} + \frac{3k}{q} \]

Therefore,

\[ \frac{R_2 W_6 L_3}{R_1 L_6 W_3} \ln \left( \frac{W_1 W_4 L_2 L_3}{L_1 L_4 W_2 W_3} \right) = \frac{q}{kT_0} (V_{\text{GO}} - V_{\text{BE0}}) - 3 \]

Under the above constraint, \( V_{\text{REF}} \) has an \( \approx \) zero \( TCF \) at \( T = T_0 \) and has a value of

\[ V_{\text{REF}} = V_{\text{GO}} + \frac{3kT}{q} \left[ 1 + \ln \left( \frac{T_0}{T} \right) \right] = V_{\text{GO}} + \frac{3kT}{q} \]

Practical values of \( \partial V_{\text{REF}} / \partial T \) for the weak inversion bandgap are less than 100 ppm/°C.
SUMMARY

Summary

- Bandgap voltage references can achieve temperature dependence less than 50 ppm/°C
- Bandgap voltage references are also independent of power supply
- Need correction circuitry to achieve smaller values of temperature coefficient