SUPPLY AND TEMPERATURE INDEPENDENT BIASING

INTRODUCTION

Objective
The objective of this presentation is:
1.) Characterize the dependence of bias circuits on the power supply
2.) Introduce circuits that have various degrees of power supply independence

Outline
• Characterization of power supply dependence
• Simple bias circuits
• Bootstrapped bias circuits
• Temperature characterization of bias circuits
• Summary
CHARACTERIZATION OF POWER SUPPLY DEPENDENCE

Characteristics of a Voltage or Current Reference

What is a Voltage or Current Reference?

A voltage or current reference is an independent voltage or current source that has a high degree of precision and stability.

Requirements of a Reference Circuit:
- Should be independent of power supply
- Should be independent of temperature
- Should be independent of processing variations
- Should be independent of noise and other interference

Fig. 4.5-1
Power Supply Independence

How do you characterize power supply independence?

Use the concept of sensitivity (we will use voltage although $I_{REF}$ can be substituted for $V_{REF}$ in the following):

$$S_{V_{REF}}^{V_{DD}} = \frac{V_{REF}/V_{REF}}{V_{DD}/V_{DD}} = \frac{V_{DD}}{V_{REF}} \left( \frac{V_{REF}}{V_{DD}} \right)$$

Application of sensitivity to determining power supply dependence:

$$\frac{V_{REF}}{V_{REF}} = \left( S_{V_{REF}}^{V_{DD}} \right) \frac{V_{DD}}{V_{DD}}$$

Thus, the fractional change in the reference voltage is equal to the sensitivity times the fractional change in the power supply voltage.

For example, if the sensitivity is 1, then a 10% change in $V_{DD}$ will cause a 10% change in $V_{REF}$.

Ideally, we want $S_{V_{REF}}^{V_{DD}}$ to be zero for power supply independence.
SIMPLE BIAS/REFERENCE CIRCUITS

Voltage References using Voltage Division

Resistor voltage divider.

\[ V_{REF} = \frac{R_2}{R_1 + R_2} V_{DD} \]

Active device voltage divider. Fig. 4.5-2

\[ V_{REF} = \frac{V_{TN} + \sqrt{\frac{\beta_P}{\beta_N}} (V_{DD} - |V_{TP}|)}{1 + \sqrt{\frac{\beta_P}{\beta_N}}} \]

\[ S_{V_{REF}}^{V_{DD}} = 1 \]

Assume \( \beta_N = \beta_P \) and \( V_{TN} = |V_{TP}| \) \( \Rightarrow \) \[ S_{V_{REF}}^{V_{DD}} = 1 \]
MOSFET-Resistance Voltage References

\[ V_{REF} = V_{GS} = V_T + \sqrt{\frac{2(V_{DD} - V_{REF})}{\beta R}} \]

or \[ V_{REF} = V_T - \frac{1}{\beta R} + \sqrt{\frac{2(V_{DD} - V_T)}{\beta R} + \frac{1}{(\beta R)^2}} \]

\[ S_{V_{REF}}^V_{DD} = \left( \frac{1}{1 + \beta (V_{REF} - V_T) R} \right) \left( \frac{V_{DD}}{V_{REF}} \right) \]

Assume that \( V_{DD} = 5\text{V}, \ W/L = 2 \) and \( R = 100\text{k}\Omega \),

Thus, \( V_{REF} \approx 1.281\text{V} \) and \( S_{V_{REF}}^V_{DD} = 0.283 \)

This circuit allows \( V_{REF} \) to be larger.

If the current in \( R_1 \) (and \( R_2 \)) is small compared to the current flowing through the transistor, then

\[ V_{REF} \approx \left( \frac{R_1 + R_2}{R_2} \right) V_{GS} \]
Bipolar-Resistance Voltage References

![Bipolar-Resistance Voltage References](image)

\[ V_{REF} = V_{EB} = \frac{kT}{q} \ln \left( \frac{I}{I_s} \right) \]

\[ I = \frac{V_{CC} - V_{EB}}{R} = \frac{V_{CC}}{R} \]

\[ V_{REF} \approx \frac{kT}{q} \ln \left( \frac{V_{CC}}{R I_s} \right) \]

\[ S_{V_{REF}}^{V_{CC}} = \frac{1}{\ln[V_{CC}/(R I_s)]=1/\ln(I/I_s)} \]

If \( V_{CC}=5V, R = 4.3k\Omega \) and \( I_s = 1fA \), then \( V_{REF} = 0.719V \).

Also, \( S_{V_{REF}}^{V_{CC}} = 0.0362 \)

If the current in \( R_1 \) (and \( R_2 \)) is small compared to the current flowing through the transistor, then

\[ V_{REF} \approx \left( \frac{R_1 + R_2}{R_1} \right) V_{EB} \]
Example 1 - Design of a Higher-Voltage Bipolar Voltage Reference

Use the circuit on the previous slide to design a voltage reference having $V_{REF} = 2.5\,\text{V}$ when $V_{CC} = 5\,\text{V}$. Assume $I_s = 1\,\text{fA}$ and $\beta_F = 100$. Evaluate the sensitivity of $V_{REF}$ with respect to $V_{CC}$.

**Solution**

Choose $I$ (the current flowing through $R$) to be $100\,\mu\text{A}$. Therefore $R = \frac{V_{CC} - V_{REF}}{100\,\mu\text{A}} = \frac{2.5\,\text{V}}{100\,\mu\text{A}} = 25\,\text{k}\Omega$.

Choose $I_1$ (the current flowing through $R_1$) to be $50\,\mu\text{A}$. Therefore the current flowing in the emitter is $50\,\mu\text{A}$. The value of $V_{EB} = V_t \ln \left( \frac{50\,\mu\text{A}}{1\,\text{fA}} \right) = 0.638\,\text{V}$.

$R_1 = \frac{0.638\,\text{V}}{50\,\mu\text{A}} = 12.76\,\text{k}\Omega$

With $50\,\mu\text{A}$ in the emitter, the base current is approximately $5\,\mu\text{A}$.

Therefore, the current through $R_2$ is $55\,\mu\text{A}$.

Since $V_{REF} = I_{R2}R_2 + 0.638\,\text{V} = 2.5\,\text{V}$, we get $R_2 = \left( \frac{2.5\,\text{V} - 0.638\,\text{V}}{55\,\mu\text{A}} \right) = 33.85\,\text{k}\Omega$.

The sensitivity of $V_{REF}$ with respect to $V_{CC}$ is

$$S_{V_{REF}/V_{CC}} = \left( \frac{R_1 + R_2}{R_1} \right) S_{V_{REF}/V_{CC}} = \left( \frac{12.76\,\text{k}\Omega + 33.85\,\text{k}\Omega}{12.76\,\text{k}\Omega} \right) \left( \frac{1}{\ln(I_Q/I_s)} \right) = 3.652(0.0406) = 0.148$$
**Breakdown Diode Voltage References**

If the power supply voltage is high enough, i.e. $V_{DD} \approx 10\text{V}$, the breakdown diode can be used as a voltage reference.

![V-I characteristics of a breakdown diode.](image)

$V_{REF} = V_{BV}$

$$S_{V_{DD}}^{V_{REF}} = \left( \frac{\partial V_{REF}}{\partial V_{DD}} \right) \left( \frac{V_{DD}}{V_{REF}} \right) \cong \frac{v_{ref}}{v_{dd}} \left( \frac{V_{DD}}{V_{BV}} \right) = \left( \frac{r_{Z}}{r_{Z} + R} \right) \left( \frac{V_{DD}}{V_{BV}} \right)$$

where $r_{Z}$ is the small-signal impedance of the breakdown diode at $I_{Q}$ (30 to 100Ω).

Typical sensitivities are 0.02 to 0.05.
Note that the temperature dependence could be zero if $V_B$ was a variable.
**BOOTSTRAPPED BIAS/REFERENCE CIRCUITS**

**Bootstrapped Current Source**

So far, none of the previous references have shown very good independence from power supply. Let us now examine a technique which does achieve the desired independence.

Circuit:

![Circuit Diagram]

**Principle:**

If M3 = M4, then \( I_1 = I_2 \). However, the M1-R loop gives

\[
V_{GS1} = V_{T1} + \sqrt{\frac{2I_1}{K_N'(W_1/L_1)}}
\]

Solving these two equations gives

\[
I_2 = \frac{V_{GS1}}{R} = \frac{V_{T1}}{R} + \left(\frac{1}{R}\right)\sqrt{\frac{2I_1}{K_N'(W_1/L_1)}}
\]

The output current, \( I_{out} = I_1 = I_2 \) can be solved as

\[
I_{out} = \frac{V_{T1}}{R} + \frac{1}{\beta_1 R^2} + \frac{1}{R} \sqrt{\frac{2V_{T1}}{\beta_1 R} + \frac{1}{(\beta_1 R)^2}}
\]
which is not dependent upon the power supply.
The current $I_D^2$ appears to be okay, why is $I_D^1$ increasing? Apparently, the channel modulation on the current mirror M3-M4 is large.

At $V_{DD} = 5\text{V}$, $V_{SD^3} = 2.83\text{V}$ and $V_{SD^4} = 1.09\text{V}$ which gives $I_D^3 = 1.067 I_D^4 \approx 107\mu\text{A}$

Need to cascode the upper current mirror.

SPICE Input File:

```plaintext
Simple, Bootstrap Current Reference
VDD 1 0 DC 5.0
VSS 9 0 DC 0.0
M1 5 7 9 N W=20U L=1U
M2 3 5 7 9 N W=20U L=1U
M3 5 3 1 1 P W=25U L=1U
M4 3 3 1 1 P W=25U L=1U
M5 9 3 1 1 P W=25U L=1U
RB 1 6 100KILOHM
.OP
.DC VDD 0 5 0.1
.MODEL N NMOS VTO=0.7 KP=110U GAMMA=0.4 +PHI=0.7 LAMBDA=0.04
.MODEL P PMOS VTO=-0.7 KP=50U GAMMA=0.57 +PHI=0.8 LAMBDA=0.05
```

ECE 4430 - Analog Integrated Circuits and Systems

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Cascoded Bootstrapped Current Source

SPICE Input File:

Cascode, Bootstrap Current Reference
VDD 1 0 DC 5.0
VSS 9 0 DC 0.0
M1 5 7 9 9 N W=20U L=1U
M2 4 5 7 9 N W=20U L=1U
M3 2 3 1 1 P W=25U L=1U
M4 8 3 1 1 P W=25U L=1U
M8 6 6 9 9 N W=1U L=1U
M7 6 6 5 9 N W=20U L=1U
RB 1 6 100KILOHM
.OP
.DC VDD 0 5 0.1
.MODEL N NMOS VTO=0.7 KP=110U
GAMMA=0.4 PHI=0.7 LAMBDA=0.04
M3C 5 4 2 1 P W=25U L=1U
MC4 3 4 8 1 P W=25U L=1U
RON 3 4 4KILOHM
M5 9 3 1 1 P W=25U L=1U
R 7 9 10KILOHM

**Base-Emitter Referenced Circuit**

![Circuit Diagram]

\[ I_{out} = I_2 = \frac{V_{EB1}}{R} \]

BJT can be a MOSFET in weak inversion.
TEMPERATURE DEPENDENCE OF BIAS CIRCUITS

Characterization of Temperature Dependence

The objective is to minimize the fractional temperature coefficient defined as,

\[ TCF = \frac{1}{V_{REF}} \left( \frac{V_{REF}}{T} \right) = \frac{1}{T} S_{V_{REF}}^{T} \text{ parts per million per } ^\circ C \text{ or ppm/} ^\circ C \]

Temperature dependence of PN junctions:

\[ I_s = K T^3 \exp \left( \frac{v}{V_t} \right) \]

\[ \frac{dV_{BE}}{dT} \approx \frac{V_{BE} - V_GO}{T} = -2mV/^\circ C \text{ at room temperature} \]

\[ V_GO = 1.205 \text{ V at room temperature and is called the bandgap voltage} \]

Temperature dependence of MOSFET in strong inversion:

\[ \frac{dv_{GS}}{dT} = \frac{dV}{dT} + \sqrt{2L \frac{d}{W C_{ox}}} \frac{d}{dT} \left( \sqrt{\frac{i_D}{\mu_o}} \right) \]

\[ \mu_o = K T^{-1.5} \]

\[ \frac{dV_{GS}}{dT} \approx -\alpha \approx -2.3 \text{ mV/^\circ C} \]

\[ V_T(T) = V_T(T_0) - \alpha(T - T_0) \]

Resistors:
\[
\frac{1}{R} \frac{dR}{dT} \text{ ppm/}^\circ\text{C}
\]
Bipolar-Resistance Voltage References

From previous work we know that,

\[ V_{REF} = \frac{kT}{q} \ln \left( \frac{V_{DD} - V_{REF}}{RI_s} \right) \]

However, not only is \( V_{REF} \) a function of \( T \), but \( R \) and \( I_s \) are also functions of \( T \).

\[
\therefore \frac{dV_{REF}}{dT} = \frac{k}{q} \ln \left( \frac{V_{DD} - V_{REF}}{RI_s} \right) + \frac{kT}{q} \left( \frac{RI_s}{V_{DD} - V_{REF}} \right) \frac{dV_{REF}}{dT} - \left( \frac{V_{DD} - V_{REF}}{RI_s} \right) \left( \frac{dR}{RdT} + \frac{dI_s}{I_sdT} \right)
\]

\[
= \frac{V_{REF}}{T} - \frac{V_t}{V_{DD} - V_{REF}} \frac{dV_{REF}}{dT} - V_t \left( \frac{dR}{RdT} + \frac{dI_s}{I_sdT} \right) = \frac{V_{REF} - V_{GO}}{T} - \frac{V_t}{V_{DD} - V_{REF}} \frac{dV_{REF}}{dT} - 3\frac{V_t}{T} - \frac{V_t dR}{R dT}
\]

\[
\therefore \frac{dV_{REF}}{dT} = \frac{V_{REF} - V_{GO}}{T} - \frac{V_t}{T} \frac{dR}{RdT} - \frac{3V_t}{T}
\]

\[
TC_F = \frac{1}{V_{REF}} \frac{dV_{REF}}{dT} = \frac{V_{REF} - V_{GO}}{V_{REF}T} - \frac{V_t}{V_{REF}RdT} - \frac{3V_t}{V_{REF}T}
\]

If \( V_{REF} = 0.6V \), \( V_t = 0.026V \), and the resistor is polysilicon, then at room temperature the \( TC_F \) is

\[
TC_F = \frac{0.6 - 1.205}{0.6 \cdot 300} - \frac{0.026 \cdot 0.0015}{0.6} - \frac{3 \cdot 0.026}{0.6 \cdot 300} = -3361 \times 10^{-6} - 65 \times 10^{-6} - 433 \times 10^{-6} = -3859 \text{ppm/°C}
\]
MOSFET Resistor Voltage Reference

From previous results we know that

\[ V_{REF} = V_{GS} = V_T + \sqrt{\frac{2(V_{DD} - V_{REF})}{\beta R}} \]

or

\[ V_{REF} = V_T - \frac{1}{\beta R} + \sqrt{\frac{2(V_{DD} - V_T)}{\beta R} + \frac{1}{(\beta R)^2}} \]

Note that \( V_{REF}, V_T, \beta, \) and \( R \) are all functions of temperature.

It can be shown that the \( TC_F \) of this reference is

\[
\frac{dV_{REF}}{dT} = -\alpha + \sqrt{\frac{V_{DD} - V_{REF}}{2\beta R} \left( \frac{1.5}{T} - \frac{1}{R \, dT} \right)}
\]

\[
1 + \frac{1}{\sqrt{2\beta R (V_{DD} - V_{REF})}}
\]

\[ TC_F = \frac{-\alpha + \sqrt{\frac{V_{DD} - V_{REF}}{2\beta R} \left( \frac{1.5}{T} - \frac{1}{R \, dT} \right)}}{V_{REF}(1 + \frac{1}{\sqrt{2\beta R (V_{DD} - V_{REF})}})} \]

Fig. 4.5-4
Example 1 - Calculation of MOSFET-Resistor Voltage Reference $TC_F$

Calculate the temperature coefficient of the MOSFET-Resistor voltage reference where $W/L=2$, $V_{DD}=5V$, $R=100k\Omega$ using the parameters of Table 3.1-2. The resistor, R, is polysilicon and has a tempco of 1500 ppm/°C.

Solution

First, calculate $V_{REF}$. Note that $\beta R = 220 \times 10^{-6} \times 10^5 = 22V^{-1}$ and $\frac{dR}{RT} = 1500 \text{ppm/°C}$

$$\therefore V_{REF} = 0.7 - \frac{1}{22} + \sqrt{\frac{2(5 - 0.7)}{22}} + \left(\frac{1}{22}\right)^2 = 1.281V$$

Now, $\frac{dV_{REF}}{dT} = \frac{-2.3 \times 10^{-3} + \sqrt{\frac{5 - 1.281}{2(22)}} \left(\frac{1.5}{300} - 1500 \times 10^{-6}\right)}{1 + \sqrt{2(22) (5 - 1.281)}} = -1.189 \times 10^{-3}\text{V/°C}$

The fractional temperature coefficient is given by

$$TC_F = -1.189 \times 10^{-3} \left(\frac{1}{1.281}\right) = -928 \text{ ppm/°C}$$
Bootstrapped Current Source/Sink

Gate-source referenced source:

The output current was given as, \( I_{\text{out}} = \frac{V_{T1}}{R} + \frac{1}{\beta_1 R^2} + \frac{1}{R} \sqrt{\frac{2V_{T1}}{\beta_1 R} + \frac{1}{(\beta_1 R)^2}} \).

Although we could grind out the derivative of \( I_{\text{out}} \) with respect to \( T \), the temperature performance of this circuit is not that good to spend the time to do so. Therefore, let us assume that \( V_{GS1} \approx V_{T1} \) which gives

\[ I_{\text{out}} \approx \frac{V_{T1}}{R} \quad \Rightarrow \quad \frac{dI_{\text{out}}}{dT} = \frac{1}{R} \frac{dV_{T1}}{dT} - \frac{1}{R^2} \frac{dR}{dT} \]

In the resistor is polysilicon, then

\[ TCF = \frac{1}{I_{\text{out}}} \frac{dI_{\text{out}}}{dT} = \frac{1}{V_{T1}} \frac{dV_{T1}}{dT} - \frac{1}{R} \frac{dR}{dT} = -\frac{-2.3 \times 10^{-3}}{0.7} - 1.5 \times 10^{-3} = -4786 \text{ppm/°C} \]

Base-emitter referenced source:

The output current was given as, \( I_{\text{out}} = I_2 = \frac{V_{BE1}}{R} \).

The \( TCF = \frac{1}{V_{BE1}} \frac{dV_{BE1}}{dT} - \frac{1}{R} \frac{dR}{dT} \)

If \( V_{BE1} = 0.6 \text{V} \) and the resistor is poly, then the \( TCF = \frac{1}{0.6}(-2 \times 10^{-3}) - 1.5 \times 10^{-3} = -4833 \text{ppm/°C} \).
Zero Temperature Coefficient (ZTC) Point for MOSFETs

For a given value of gate-source voltage, the drain current of the MOSFET will be independent of temperature.

Consider the following circuit:

Assume that the transistor is saturated and that:

\[ \mu = \mu_0 \left( \frac{T}{T_0} \right)^{1.5} \quad \text{and} \quad V_T(T) = V_T(T_0) + \alpha(T-T_0) \]

where \( \alpha = -0.0023\,\text{V/°C} \) and \( T_0 = 27^\circ\text{C} \)

\[ I_D(T) = \frac{\mu_0 C_{ox} W}{2L} \left( T \right) \left[ V_{GS} - V_{T0} - \alpha(T-T_0) \right]^2 \]

\[ \frac{dI_D}{dT} = \frac{-1.5 \mu_0 C_{ox}}{2T_0} \left( T \right)^{2.5} \left[ V_{GS} - V_{T0} - \alpha(T-T_0) \right]^2 + \alpha \mu_0 C_{ox} \left( T \right)^{1.5} \left[ V_{GS} - V_{T0} - \alpha(T-T_0) \right] = 0 \]

\[ V_{GS} - V_{T0} - \alpha(T-T_0) = \frac{4T\alpha}{3} \quad \Rightarrow \quad V_{GS}(\text{ZTC}) = V_{T0} - \alpha T - \frac{\alpha T}{3} \]

Let \( K' = 10\,\mu\text{A/V}^2 \), \( W/L = 5 \) and \( V_{T0} = 0.71\,\text{V} \).

At \( T = 27^\circ\text{C} \) (300°K), \( V_{GS}(\text{ZTC}) = 0.71 - (-0.0023)(300°K) - (0.333)(-0.0023)(300°K) = 1.63\,\text{V} \)

At \( T = 27^\circ\text{C} \) (300°K), \( I_D = (10\,\mu\text{A/V}^2)(5/2)(1.63-0.71)^2 = 21.2\,\mu\text{A} \)

At \( T = 200^\circ\text{C} \) (473°K), \( V_{GS}(\text{ZTC}) = 0.71 - (-0.0023)(300°K) - (0.333)(-0.0023)(473°K) = 1.76\,\text{V} \)
Experimental Verification of the ZTC Point

The data below is for a 5µm n-channel MOSFET with W/L = 50µm/10µm, \( N_A = 10^{-16}\text{cm}^{-3} \), \( t_{ox} = 650\text{Å} \), \( \mu_C \)ox = 10µA/V², and \( V_{T0} = 0.71\text{V} \).
ZTC Point for PMOS

The data below is for a 5µm p-channel MOSFET with $W/L = 50\mu m/10\mu m$, $N_D = 2 \times 10^{-15} \text{cm}^{-3}$, and $t_{ox} = 650\text{Å}$. 

Zero temperature coefficient will occur for every MOSFET up to about 200°C.
SUMMARY OF POWER SUPPLY INDEPENDENT BIASING

Summary

- Reasonably good, simple references are possible
- Best power supply sensitivity is approximately 0.01
  (10% change in power supply causes a 0.1% change in reference)
- Typical simple reference temperature dependence is $\approx 1000$ ppm/°C
- Can obtain zero temperature coefficient over a limited range of operation

<table>
<thead>
<tr>
<th>Type of Reference</th>
<th>$S_{\frac{V_{REF}}{V_{DD}}}$</th>
<th>$TC_F$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage division</td>
<td>1</td>
<td>Good if matched</td>
<td></td>
</tr>
<tr>
<td>MOSFET-R</td>
<td>$&lt;1$</td>
<td>$&gt;1000$ ppm/°C</td>
<td></td>
</tr>
<tr>
<td>BJT-R</td>
<td>$&lt;&lt;1$</td>
<td>$&gt;1000$ ppm/°C</td>
<td>Requires start-up circuit</td>
</tr>
<tr>
<td>Bootstrap Gate-Source Referenced</td>
<td>Good if currents are matched</td>
<td>$&gt;1000$ ppm/°C</td>
<td>Requires start-up circuit</td>
</tr>
<tr>
<td>Bootstrap Base-emitter Referenced</td>
<td>Good if currents are matched</td>
<td>$&gt;1000$ ppm/°C</td>
<td>Requires start-up circuit</td>
</tr>
</tbody>
</table>