### 3.6 - DEVICE MISMATCH IN DIFFERENTIAL AMPLIFIERS

#### INTRODUCTION

**Objective**
The objective of this presentation is:
1.) Characterize the dependence of bias circuits on the power supply
2.) Introduce circuits that have various degrees of power supply independence

**Outline**
- Characterization of power supply dependence
- Simple bias circuits
- Bootstrapped bias circuits
- Temperature characterization of bias circuits

**Objective**
The objective of this presentation is:
1.) Illustrate the method of analyzing mismatches
2.) Analyze the input current and voltage offsets for differential amplifiers

**Outline**
- The general approach to analyzing mismatches
- Input voltage and current offsets of BJT differential amplifiers
- Input voltage offsets of MOS differential amplifiers
- Summary

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### MISMATCH ANALYSIS METHODS

**General Method**
Suppose that two performances, \( p_1 \) and \( p_2 \), can be written as
\[
p_1 = f_1(x_1, y_1, z_1, \ldots) \quad \text{and} \quad p_2 = f_2(x_2, y_2, z_2, \ldots)
\]
Ideally, \( y_1 \) should be equal to \( y_2 \), but in practice their difference could be expressed as
\[
\text{Error} = e(p_1, p_2) = f(x_1, y_1, z_1, \ldots, x_2, y_2, z_2, \ldots)
\]
Now assume that \( x_1, y_1, z_1, \ldots, x_2, y_2, z_2, \ldots \) can be expressed in terms of their difference and average values. We illustrate only for \( x_1 \) and \( x_2 \),
\[
\Delta x = x_1 - x_2 \quad \text{and} \quad x = \frac{x_1 + x_2}{2}
\]
We can solve for \( x_1 \) and \( x_2 \) in terms of \( \Delta x \) and \( x \) as follows,
\[
x_1 = x + 0.5 \Delta x \quad \text{and} \quad x_2 = x - 0.5 \Delta x
\]
Now the error can be express as
\[
e(p_1, p_2) = f(x + 0.5 \Delta x, x - 0.5 \Delta x)
\]
This expressing can generally be simplified by assuming that \( \Delta x \ll x \) and using the following approximations,
\[
\frac{1}{1+\epsilon} = 1+\epsilon \quad \text{or} \quad \frac{1}{1+\epsilon} = 1-\epsilon
\]
and neglecting higher power values of \( \epsilon \), i.e. \( \epsilon^2 \)
INPUT VOLTAGE AND CURRENT OFFSETS OF THE BJT DIFFERENTIAL AMPLIFIER

Model for Input Offset Voltage and Current

where

\[ V_{OS} = V_{BE1} - V_{BE2} = V_{BE1} \left( \frac{I_{C1}}{I_{s1}} \right) - V_{BE2} \left( \frac{I_{C2}}{I_{s2}} \right) = V_{BE1} \left( \frac{I_{C1}}{I_{s1}} \right) - V_{BE2} \left( \frac{I_{C2}}{I_{s2}} \right) \]

Substituting these values into the expression for \( V_{OS} \), we get

\[ V_{OS} = V_{BE1} \left( \frac{I_{C1}}{I_{s1}} \right) - V_{BE2} \left( \frac{I_{C2}}{I_{s2}} \right) \]

Combining these two relationships gives

\[ V_{OS} = V_{BE1} \left( \frac{I_{C1}}{I_{s1}} \right) - V_{BE2} \left( \frac{I_{C2}}{I_{s2}} \right) \]

How does \( I_s \) depend upon the semiconductor parameters?

\[ I_s = \frac{q n_i^2 D_n}{N_A W B_1(V_{CB})} A_1 = \frac{q n_i^2 D_n}{Q B_1(V_{CB})} A_1 \quad \text{and} \quad I_s = \frac{q n_i^2 D_n}{N_A W B_2(V_{CB})} A_2 = \frac{q n_i^2 D_n}{Q B_2(V_{CB})} A_2 \]

where \( W_B(V_{CB}) \) is the base width as a function of \( V_{CB} \), \( N_A \) is the acceptor density in the base and \( A \) is the emitter area.

BJT Input Offset Voltage - Continued

In order for the output voltage to be zero, it is required that

\[ I_{C1} R_C = I_{C2} R_C \quad \Rightarrow \quad \frac{I_{C1}}{I_{C2}} = \frac{R_C}{R_C} \]

Combining these two relationships gives

\[ V_{OS} = V_t \ln \left[ \frac{R_C}{R_C} \right] \left( \frac{A_2}{A_1} \right) \left( \frac{Q B_1(V_{CB})}{Q B_2(V_{CB})} \right) \]

Making the following definitions,

\[ \Delta R_C = R_C - R_C, \quad \Delta A = A_2 - A_1 = A_2 - A_1 = A_2 - A_1, \quad \Delta Q_B = Q B_1 - Q B_2, \quad \text{and} \quad Q B = \frac{Q B_1 + Q B_2}{2} \]

which gives

\[ R_C = \frac{\Delta R_C}{2}, \quad R_C = \frac{\Delta R_C}{2}, \quad \Delta A = A_2 - A_1, \quad \Delta Q_B = Q B_1 - Q B_2, \quad \text{and} \quad Q B = \frac{Q B_1 + Q B_2}{2} \]

Substituting these values into the expression for \( V_{OS} \) gives,

\[ V_{OS} = V_t \ln \left[ \frac{R_C - \Delta R_C}{R_C + \Delta R_C} \right] \left( \frac{A - \Delta A}{A + \Delta A} \right) \left( \frac{Q B + \Delta Q B}{Q B - \Delta Q B} \right) = V_t \ln \left[ 1 - \frac{\Delta R_C}{R_C} \right] \left( 1 - \frac{\Delta A}{A} \right) \left( 1 + \frac{\Delta Q B}{Q B} \right) \]

if \( \Delta R_C \ll R_C \), \( \Delta A \ll A \), and \( \Delta Q_B \ll Q_B \)

Expanding the logarithm and neglecting higher order terms gives

\[ V_{OS} = V_t \left( - \frac{\Delta R_C}{R_C} - \frac{\Delta A}{A} + \frac{\Delta Q_B}{Q B} \right) = V_t \left( - \frac{\Delta R_C}{R_C} - \frac{\Delta A}{A} - \frac{\Delta Q_B}{Q B} \right) \]

where

\[ \frac{\Delta A}{A} = \frac{\Delta Q_B}{Q B} \]
Example 1 - Calculation of Input Voltage Offset for a BJT Differential Amplifier

Find the value of the input offset voltage for a BJT differential amplifier at room temperature if the
standard deviations of the resistor match and saturation current match are 1% and 5%, respectively.
Assume that the standard deviations are correlated. Repeat this example if the standard deviations are not
correlated.

Solution

For the correlated case we have,

\[ V_{OS} = V_t \left( \frac{\Delta R_C}{R_C} - \frac{\Delta I_s}{I_s} \right) = 0.026(-0.01-0.05) = -1.5\text{mV} \]

(Magnitude, not sign is important since the polarity of the mismatch is not known.)

When the variation in \( R_C \) and \( I_s \) are uncorrelated then we get,

\[ V_{OS} = V_t \sqrt{\left( \frac{\Delta R_C}{R_C} \right)^2 + \left( \frac{\Delta I_s}{I_s} \right)^2} = 0.026\sqrt{(0.01)^2+(0.05)^2} = (0.026)0.051 = 1.3\text{mV} \]

Temperature Dependence of the Input Offset Voltage

The temperature dependence of \( V_{OS} \) is found by examining the temperature dependence of

\[ V_{OS} = V_t \left( \frac{\Delta R_C}{R_C} - \frac{\Delta I_s}{I_s} \right) \]

While \( I_s \) and \( R_C \) have reasonably large temperature dependence, the temperature dependence of their
difference can be neglected. Therefore assuming a value of \( V_{OS} = 2\text{mV} \), we get

\[ \frac{dV_{OS}}{dT} = \frac{V_{OS}}{T} = \frac{2\text{mV}}{300} = 6.67\mu\text{V/°C} \]

Offset voltage can be cancelled using external circuitry but to cancel the temperature drift requires the
external circuitry to have the same temperature dependence.
**Input Offset Current of the BJT Differential Amplifier**

Consider the following model based on the previous circuit,

\[ V_{CC} \quad V_{EE} \quad Q1 \quad Q2 \quad I_{EE} \]

\[ V_{OD} \quad R_C \quad R_C \quad i_{C1} \quad i_{C2} \quad v_{BE1} \quad v_{BE2} \quad i_{B1} \quad i_{B2} \quad I_{OS} \quad + \]

\[ i_{B1} \quad - \quad v_{BE1} \quad + \quad v_{BE2} \quad i_{B2} \quad - \quad I_{OS} \quad 2 \]

The input offset current of the BJT differential amplifier can be written as,

\[ I_{B1} = I_B + 0.5I_{OS} \quad \text{and} \quad I_{B2} = I_B - 0.5I_{OS} \quad \Rightarrow \quad I_{OS} = I_{B2} - I_{B1} = \frac{I_{C2}}{\beta_{F2}} - \frac{I_{C1}}{\beta_{F1}} \]

Defining \( \Delta I_C = I_{C2} - I_{C1} \), \( I_C = \frac{I_{C1} + I_{C2}}{2} \), \( \Delta \beta_F = \beta_{F2} - \beta_{F1} \), and \( \beta_F = \frac{\beta_{F1} + \beta_{F2}}{2} \)

which gives

\[ I_{C1} = I_C - \frac{\Delta I_C}{2}, \quad I_{C2} = I_C + \frac{\Delta I_C}{2}, \quad \beta_{F1} = \beta_F - \frac{\Delta \beta_F}{2}, \quad \text{and} \quad \beta_{F2} = \beta_F + \frac{\Delta \beta_F}{2} \]

The input offset current of the BJT differential amplifier can be written as,

\[ I_{OS} = I_{C1} \frac{\Delta I_C}{2} + I_C \frac{\Delta \beta_F}{2} = I_C \left( \frac{\Delta I_C}{2} \cdot \frac{\Delta \beta_F}{\beta_F} \right) \quad \text{when} \quad \Delta I_C \ll I_C \quad \text{and} \quad \Delta \beta_F \ll \beta_F \]

Recalling that for the output voltage to be zero that

\[ \frac{I_{C1}}{I_{C2}} = \frac{R_{C2}}{R_{C1}} \]

then

\[ \frac{1 - \frac{\Delta I_C}{2I_C}}{1 + \frac{\Delta R_C}{2R_C}} \Rightarrow \left( 1 - \frac{\Delta I_C}{2I_C} \right) = \left( 1 - \frac{\Delta R_C}{2R_C} \right) \Rightarrow \frac{\Delta I_C}{I_C} = \frac{\Delta R_C}{R_C} \]

Therefore,

\[ I_{OS} = I_C \frac{\Delta R_C}{\beta_F R_C} \frac{\Delta \beta_F}{\beta_F} = I_C \left( \frac{\Delta R_C}{R_C} \frac{\Delta \beta_F}{\beta_F} \right) \]

Typically \( \Delta \beta_F/\beta_F \) is about 10% and \( \Delta R_C/R_C \) is about 1% giving

\[ I_{OS} = I_C \left( 0.01 + 0.1 \right) = 0.11I_B = 1.1 \mu A \] assuming that \( I_B = 10 \mu A \)
INPUT VOLTAGE OFFSET OF MOS DIFFERENTIAL AMPLIFIER

Model for Input Offset Voltage

\[ V_{OS} = V_{GS1} - V_{GS2} = \sqrt{\frac{2I_D}{\beta_1}} + V_{T1} - \sqrt{\frac{2I_D}{\beta_2}} + V_{T2} \]

where for \( v_{in} = 0 \),

\[ V_{OS} = V_{GS1} - V_{GS2} = \frac{2I_D}{\beta_1} + V_{T1} - \frac{2I_D}{\beta_2} + V_{T2} \]

Define,

\[ \Delta I_D = I_{D1} - I_{D2}, \quad \Delta \beta = \beta_1 - \beta_2, \quad \Delta V_T = V_{T1} - V_{T2} \]

Gives,

\[ I_{D1} = I_D + \frac{\Delta I_D}{2}, \quad I_{D2} = I_D - \frac{\Delta I_D}{2} \]

\[ \Delta \beta = \beta_1 - \beta_2, \quad \Delta V_T = V_{T1} - V_{T2} \]
Example 2 - Calculation of Input Voltage Offset for a MOS Differential Amplifier

Find the value of the input offset voltage for a MOS differential amplifier at room temperature if the standard deviations of the resistor match and beta match are 1% and 5%, respectively. Assume that the threshold voltage deviation is normalized to the value of $V_{GS} - V_T = 1.5V$ and is 10%. Assume that the standard deviations are correlated. Repeat this example if the standard deviations are not correlated.

**Solution**

For the correlated case we have,

$$V_{OS} = \frac{\Delta V_T}{V_{GS} - V_T} \cdot \frac{\Delta R_D}{2R_D} \cdot \frac{\Delta \beta}{2\beta} = \frac{0.10}{1.5} \cdot \frac{0.01}{2} \cdot \frac{0.05}{2} = 0.0367$$

$\therefore$ 

$V_{OS} = 0.0367 \times 1.5V = 55mV$

When the variation in $R_D$, $\beta$ and $V_T$ are uncorrelated then we get,

$$V_{OS} = \sqrt{\left(\frac{\Delta V_T}{V_{GS} - V_T}\right)^2 + \left(\frac{\Delta R_D}{2R_D}\right)^2 + \left(\frac{\Delta \beta}{2\beta}\right)^2} = 0.0714$$

$\therefore$ 

$V_{OS} = 0.0714 \times 1.5V = 107mV$

Temperature Dependence of the MOS Input Offset Voltage

$$V_{OS} = \Delta V_T - \sqrt{\frac{2I_D}{\beta} \left[ \frac{\Delta R_D}{2R_D} + \frac{\Delta \beta}{2\beta} \right]}$$

While $R_D$ and $V_T$ have a strong temperature dependence, the temperature dependence of there matching can be ignored.

The temperature dependence of $\beta$ is

$$\beta(T) = \frac{K'(T)W}{L} - \beta(T_0) \left( \frac{T}{T_0} \right)^{1.5} \Rightarrow \frac{d\beta}{dT} = -1.5 \beta(T_0) \left( \frac{T}{T_0} \right)^{-1.5} = \frac{-1.5 \beta(T)}{T}$$

$$\frac{dV_{OS}}{dT} = \frac{d}{dT} \left[ \frac{2I_D}{\beta} \left[ \frac{\Delta R_D}{2R_D} + \frac{\Delta \beta}{2\beta} \right] \right] = \frac{-1}{2\beta} \left[ \sqrt{\frac{2I_D}{\beta} \left[ \frac{\Delta R_D}{2R_D} + \frac{\Delta \beta}{2\beta} \right]} \right] \frac{d\beta}{dT} = \frac{3}{4T} \sqrt{\frac{2I_D}{\beta} \left[ \frac{\Delta R_D}{2R_D} + \frac{\Delta \beta}{2\beta} \right]}$$

At room temperature and a current of 100µA and $\beta = 200$, we get,

$$\frac{dV_{OS}}{dT} = \frac{1 \cdot 3 \left[ 0.01 \frac{0.05}{2} \right]}{4 \cdot 300^2} = 75\mu V/°C$$
Comparison of BJT and MOS Offset Voltages

BJT:

\[ V_{OS}(BJT) \propto V_i = \frac{I_C}{g_m(BJT)} \]

MOS:

\[ V_{OS}(MOS) \propto V_{GS} - V_T = \left( \frac{V_{GS} - V_T}{V_{GS} - V_T} \right)^2 = \frac{2I_D}{\beta} \frac{V_{GS} - V_T}{V_{GS} - V_T} = 0.5 \beta (V_{GS} - V_T) = \frac{I_D}{g_m(MOS)} \]

Assuming that the bias currents are the same, the offsets are greater for the MOS differential amplifier because the transconductances are smaller.

What is the key concept here?

*When amplifier imperfections (offset, noise, nonlinearity, etc.) are reflected to the input of the amplifier, the larger the gain of the amplifier, the smaller the reflected imperfections.*

SUMMARY

- Mismatch analysis between two components can be performed by defining a difference and average value of the components and expressing the mismatch in terms of the difference and average value.
- BJT differential amplifiers have both a voltage and current input offset.
- Cancellation of the input offset is difficult because of temperature dependence of the offset.
- The MOS differential amplifier has only input voltage offset but this offset is larger than the BJT differential amplifier.
- The temperature dependence of the MOSFET input offset voltage is about 10 times that of the BJT.
- To reduce the value of the amplifier imperfection reflected to the input of the amplifier, make the gain of the amplifier as large as possible.