3.4 - BJT DIFFERENTIAL AMPLIFIERS

INTRODUCTION

Objective
The objective of this presentation is:
1.) Define and characterize the differential amplifier
2.) Show the large-signal and small-signal performance
3.) Show alternate implementations of the differential amplifier

Outline
• Characterization and definitions
• Large-signal transconductance
• Large-signal voltage transfer
• Small-signal performance
• Other characteristics of the differential amplifier
• Summary
CHARACTERIZATION AND DEFINITIONS

What is a Differential Amplifier?

A differential amplifier is an amplifier that amplifies the difference between two voltages and rejects the average or common mode value of the two voltages.

Symbol for a differential amplifier:

\[ \begin{align*}
+ & \quad + \\
\text{v}_1 & \quad - \\
\text{v}_2 & \quad + \\
& \quad \text{v}_{\text{OUT}} \\
\end{align*} \]

\[ \text{Fig. 5.2-1A} \]

Differential and common mode voltages:

- \( \text{v}_1 \) and \( \text{v}_2 \) are called single-ended voltages. They are voltages referenced to ac ground.
- The differential-mode input voltage, \( \text{v}_{\text{ID}} \), is the voltage difference between \( \text{v}_1 \) and \( \text{v}_2 \).
- The common-mode input voltage, \( \text{v}_{\text{IC}} \), is the average value of \( \text{v}_1 \) and \( \text{v}_2 \).

\[ \begin{align*}
\text{v}_{\text{ID}} &= \text{v}_1 - \text{v}_2 \\
\text{v}_{\text{IC}} &= \frac{\text{v}_1 + \text{v}_2}{2} \\
\end{align*} \]

\[ \Rightarrow \quad \text{v}_1 = \text{v}_{\text{IC}} + 0.5 \text{v}_{\text{ID}} \quad \text{and} \quad \text{v}_2 = \text{v}_{\text{IC}} - 0.5 \text{v}_{\text{ID}} \]

\[ \begin{align*}
\text{v}_{\text{OUT}} &= A_{\text{VD}} \text{v}_{\text{ID}} \pm A_{\text{VC}} \text{v}_{\text{IC}} = A_{\text{VD}} (\text{v}_1 - \text{v}_2) \pm A_{\text{VC}} \left( \frac{\text{v}_1 + \text{v}_2}{2} \right) \\
\end{align*} \]

where

\[ A_{\text{VD}} = \text{differential-mode voltage gain} \]
\[ A_{\text{VC}} = \text{common-mode voltage gain} \]
**Differential Amplifier Definitions**

- **Common mode rejection ratio (CMRR)**

\[ CMRR = \frac{|A_{VD}|}{|A_{VC}|} \]

*CMRR* is a measure of how well the differential amplifier rejects the common-mode input voltage in favor of the differential-input voltage.

- **Input common-mode range (ICMR)**

The input common-mode range is the range of common-mode voltages over which the differential amplifier continues to sense and amplify the difference signal with the same gain.

Typically, the *ICMR* is defined by the common-mode voltage range over which all MOSFETs remain in the saturation region and all BJTs remain in the active region.

- **Output offset voltage (\(V_{OS}(\text{out})\))**

The output offset voltage is the voltage which appears at the output of the differential amplifier when the input terminals are connected together.

- **Input offset voltage (\(V_{OS}(\text{in}) = V_{OS}\))**

The input offset voltage is equal to the output offset voltage divided by the differential voltage gain.

\[ V_{OS} = \frac{V_{OS}(\text{out})}{A_{VD}} \]
LARGE-SIGNAL TRANSCONDUCTANCE

Transconductance Characteristic of the Differential Amplifier

Consider the following NPN-BJT differential amplifier (sometimes called an emitter-coupled pair):

Large-Signal Analysis:

1.) Input loop eq.: \[ v_{I1} - v_{BE1} + v_{BE2} - v_{I2} = v_{I1} - v_{I2} - v_{BE1} + v_{BE2} = v_{ID} - v_{BE1} + v_{BE2} = 0 \]

2.) Forward-active region: \[ v_{BE1} = V_t \ln \left( \frac{i_{C1}}{I_{S1}} \right) \quad \text{and} \quad v_{BE2} = V_t \ln \left( \frac{i_{C2}}{I_{S2}} \right) \]

3.) If \( I_{S1} = I_{S2} \) then \[ \frac{i_{C1}}{i_{C2}} = \exp \left( \frac{v_{I1} - v_{I2}}{V_t} \right) = \exp \left( \frac{v_{ID}}{V_t} \right) \]

4.) Nodal current equation at the emitters: \[ -(i_{E1} + i_{E2}) = I_{EE} = \frac{1}{\alpha_F} (i_{C1} + i_{C2}) \]

5.) Combining the above equations gives: \[ i_{C1} = \frac{\alpha_F I_{EE}}{1 + \exp \left( \frac{-v_{ID}}{V_t} \right)} \quad \text{and} \quad i_{C2} = \frac{\alpha_F I_{EE}}{1 + \exp \left( \frac{v_{ID}}{V_t} \right)} \]
Transconductance Characteristic of the Differential Amplifier - Continued

Plotting the collect current as a function of $v_{ID}$:

Transconductance:

\[
g_m^1 = \left. \frac{\partial i_{C1}}{\partial v_{ID}} \right|_{Q} = \frac{\alpha_F I_{EE}}{1 + \exp\left(\frac{-v_{ID}}{V_t}\right)^2 V_t} = \frac{\alpha_F I_{EE}}{4V_t} \approx \frac{I_{C1}}{2V_t} \quad \text{when } V_{ID} = 0
\]

\[
g_m^2 = \left. \frac{\partial i_{C2}}{\partial v_{ID}} \right|_{Q} = \frac{-\alpha_F I_{EE}}{1 + \exp\left(\frac{v_{ID}}{V_t}\right)^2 V_t} = \frac{-\alpha_F I_{EE}}{4V_t} = \frac{-I_{C2}}{2V_t} \quad \text{when } V_{ID} = 0
\]
VOLTAGE TRANSFER CHARACTERISTICS

Large-Signal Voltage Transfer Function

Assume load resistors as shown:

\[ v_{OD} = v_{O1} - v_{O2} = V_{CC} \cdot i_{C1}R_C - V_{CC} + i_{C2}R_C \]
\[ = R_C(i_{C2} - i_{C1}) \]

Substituting in the previous expressions and using hyperbolic trig identities gives

\[ v_{OD} = \alpha_F I_{EE} R_C \tanh \left( \frac{-v_{ID}}{2V_t} \right) \]

Illustration →
Emitter Degeneration of the BJT Differential Amplifier

Increases the range over which the emitter-coupled pair behaves as a linear amplifier with lower gain at the cost of lower gain.

We know that,

\[ i_D = i_{C1} - i_{C2} = \alpha_F I_{EE} \tanh \left( \frac{v_{ID} - (R_E i_D / \alpha_F)}{2V_t} \right) \approx \alpha_F I_{EE} \left[ v_{ID} - \frac{(R_E i_D / \alpha_F)}{2V_t} \right] \]

Solving for \( i_D \) gives,

\[ i_D = i_{C1} - i_{C2} = \frac{\alpha_F I_{EE} v_{ID}}{2V_t + R_E I_{EE}} \]

\[ g_m(DC) = \frac{di_D}{dv_{ID}} = \frac{(\alpha_F I_{EE})}{2V_t + R_E I_{EE}} = \frac{(\alpha_F I_{EE})/2V_t}{1 + R_E I_{EE}/2V_t} \]
SMALL-SIGNAL PERFORMANCE

Differential and Common-mode Small-Signal Performance

The small-signal performance of a differential amplifier can be separated into a differential mode and common mode analysis. This separation allows us to take advantage of the following simplifications.

Half-Circuit Concept:

Differential Mode Analysis

Common Mode Analysis

Note: The half-circuit concept is valid as long as the resistance seen looking into each emitter is the same.
**SMALL-SIGNAL PERFORMANCE**

**Differential and Common-mode Small-Signal Performance**

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Half-Circuit Concept:

Note: The half-circuit concept is valid as long as the resistance seen looking into each emitter is the same.
Small-Signal, Common Mode Performance of the BJT Differential Amplifier

Circuit and small-signal model:

Half-circuit performance:

\[ R_{in1} = r_{\pi 1} + (1 + \beta_{o1})2R_EE, \quad R_{out1} = r_o \left[ 1 + \frac{\beta_{o1}2R_EE}{2R_EE + r_{\pi 1}} \right] + 2R_EE || r_{\pi 1}, \quad \text{and} \quad \frac{v_{o1}}{v_{ic}} = - \frac{\beta_{o1}R_C}{r_{\pi 1} + (1 + \beta_{o1})2R_EE} \]

Common mode performance:

\[ R_{ic} = \frac{r_{\pi 1} + (1 + \beta_{o1})2R_EE}{2}, \quad R_{oc} = r_o \left[ 1 + \frac{\beta_{o1}2R_EE}{2R_EE + r_{\pi 1}} \right] + 2R_EE || r_{\pi 1}, \quad \text{and} \quad \frac{v_{oc}}{v_{ic}} = - \frac{\beta_{o1}R_C}{r_{\pi 1} + (1 + \beta_{o1})2R_EE} \]

where \( g_{m1} = g_{m2}, \ r_{\pi 1} = r_{\pi 2} \) and \( \beta_{o1} = \beta_{o2} \).
Common Mode Rejection Ratio (CMRR)

The common mode rejection ratio is a measure of the differential amplifier’s ability to reject the common mode signal and amplify the differential mode signal.

\[
\text{CMRR} = \left| \frac{A_{dm}}{A_{cm}} \right| = \left| \frac{v_o/v_{id}}{v_o/v_{ic}} \right| = \frac{\frac{g_{m1}R_C}{2\beta_o R_C}}{r_{\pi1}+(1+\beta_o)2R_{EE}} \approx g_{m1}R_{EE} = \frac{I_{EE}R_{EE}}{2V_t}
\]

Thus, the larger the input transconductance or \(R_{EE}\), the larger the common mode rejection ratio.
OTHER CHARACTERISTICS OF THE DIFFERENTIAL AMPLIFIER

Input Common Mode Voltage Range

The input common mode voltage range (ICMR) is the range of common mode input voltages over which the differential amplifier amplifies the differential signal without significant change.

Consider the following:

Maximum Input Common Mode Voltage:

\[ v_{ic}(\text{max}) = V_{CC} - 0.5I_{EE}R_C - v_{CE1}(\text{sat}) + V_{BE1} \]

Minimum Input Common Mode Voltage:

\[ v_{ic}(\text{min}) = V_{EE} + v_{CE3}(\text{sat}) + V_{BE1} \]
Slew Rate of the BJT Differential Amplifier

Slew rate is a voltage rate limit due to the fact that the current available to charge a capacitor is constant.

\[
\text{Slew rate} = \text{SR} = \frac{dv_C}{dt} = \frac{i_C}{C}
\]

where \(i_C\) and \(v_C\) are the current through and voltage across a capacitor \(C\).

A differential amplifier that has a capacitive load will experience slew rate which is seen as follows:

Note that the current in Q1 is 0 and Q2 is \(I_{EE}\). Therefore, the initial value of \(i_{Co}\) is \(i_{Co} = 0.5I_{EE} - i_{Cs}\).

If we define the slew rate across \(C_o\) as \(SR = \frac{i_{Co}}{C_o}\), then \(i_{Cs} = 0.5SR\cdot C_s = \frac{i_{Co}}{2C_o}C_s\), i.e. \(\frac{dv_{C1}}{dt} = 0.5\frac{dv_{OD}}{dt}\)

\[
\therefore SR = \frac{I_{EE}}{2C_o} - \frac{i_{Cs}}{C_o} = \frac{I_{EE}}{2C_o} - \frac{SR\cdot C_s}{2C_o} \Rightarrow SR \left(1 + \frac{C_s}{2C_o}\right) = \frac{I_{EE}}{2C_o} \Rightarrow \frac{SR}{C_o + 0.5C_s} = \frac{I_{EE}}{2C_o + C_s}
\]
SUMMARY

- A differential amplifier amplifies the difference signal between two voltages and rejects the common mode signal.
- The transconductance characteristics of the BJT differential amplifier switches from all of the current in one side to the other side within ±100mV.
- The large-signal differential voltage transfer function has the form of hyperbolic tangent.
- Emitter degeneration increases the range over which the differential amplifier behaves as a linear amplifier.
- The half circuit concept is very useful for analyzing the small-signal differential and common mode performance.
- The maximum and minimum input common mode range is:
  \[ v_{ic}(\text{max}) = V_{CC} - 0.5I_{EE}R_C - v_{CE1}(\text{sat}) + V_{BE1} \]
  \[ v_{ic}(\text{min}) = V_{EE} + v_{CE3}(\text{sat}) + V_{BE1} \]
- The differential amplifier has a slew rate limit of \( I_{EE}/C_{eq} \) where \( C_{eq} \) is the capacitance seen to ground from either collector.